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FORECASTING TROPICAL CYCLONE RECURVATURE USING AN EMPIRICAL OTHOGONAL FUNCTION REPRESENTATION OF VORTICITY FIELDS

by

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Forecasting Tropical Cyclone
Recurvature using an
Empirical Othogonal Function
Representation of Vorticity Fields

by

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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

An empirical orthogonal function (EOF) representation of relative vorticity is used to forecast recurvature (change in storm heading from west to east of 000° N) of western North Pacific tropical cyclones. The time-dependent coefficients of the first and second EOF eigenvectors vary in a systematic manner as the tropical cyclone recurves around the subtropical ridge and tend to cluster about the same values at recurvature time. In contrast, the coefficients for straight-moving storms tend to cluster in a different region in EOF space. Exploiting this Euclidean distance approach, additional EOF coefficients are identified that best represent the vorticity fields of recurving and straight-moving storms. Classification of an individual case is then into the closest time-to-recurvature in 12-h intervals or straight-moving storm category as measured in multidimensional EOF space. Although rather subjective, the Euclidean method demonstrates skill relative to climatological forecasts. A more objective discriminant analysis technique is also tested. A final version that involves the first six EOF coefficients of the 250 mb vorticity field is useful (72% correct) in identifying recurvers or straight-movers during the 72-h forecast period. Skill in classifying situations within 12-h time-to-recurvature groups is low, but might be improved using other analysis techniques or in combination with other predictors.

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I. INTRODUCTION

Tropical cyclones have formidable destructive power and annually exact tremendous losses in lives and property. The western North Pacific Ocean is the most active tropical cyclone basin in the world. An average of 31 tropical cyclones have occurred annually during the 25-year period ending in 1984 (ATCR 1984). The damage from these storms can be minimized only through preparedness and avoidance. Precautionary measures can require considerable time. Therefore, accurate storm forecasts are critically important to both the military and civilian communities.

A. BACKGROUND

Tropical cyclones can be classified into three broad categories based on their track. If a storm moves west or northwest throughout its life, it is classified as a straight-mover (TY Agnes in Fig. 1). A storm that turns from a westward or northwestward path through North to a northeastward track is defined as a recurver (ST Vanessa in Fig. 1). Storms that do not fit either the straight-mover or the recurver categories are classified as odd-movers (ST Bill in Fig. 1). Odd-mover tracks are typically erratic and may display loops or a stairstep-type track. The largest forecast errors occur when recurving storms had been forecast to move straight toward the west or northwest, or when straight-movers had been forecast to recurve to the north or northeast. Incorrect recurvature forecasts result in 72-h track forecast errors of over 1850 km (1000 n mi) almost every year (Sandgathe 1987). Situations associated with recurvature, due either to cyclone-midlatitude trough interaction or to cyclone-subtropical ridge interaction, are listed among the Joint Typhoon Warning Center's (JTWC's) most difficult forecast problems (Sandgathe 1987). Since nearly half of all western North Pacific tropical cyclones eventually recurve, these recurvature forecast questions are frequently faced by operational forecasters.

None of the present objective forecast aids in operational use are specifically designed to identify recurvature situations. Leftwich (1979) and Lage (1982) used regression analysis techniques to predict recurvature, which they defined as a net displacement north of 315° during the forecast period. Leftwich (1979) included position, motion, intensity and 1000-100 mb geopotential height predictors to forecast the probability of recurvature in Atlantic tropical cyclones. Geopotential heights were represented by gridpoint values on a relocatable storm-centered grid. Leftwich concluded

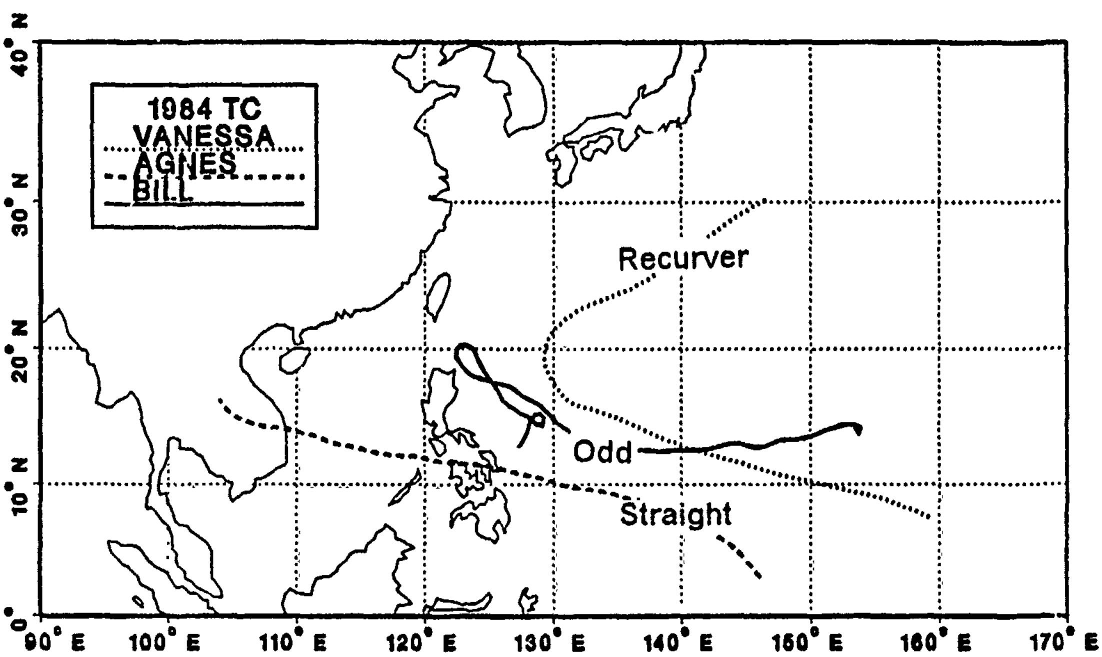

Fig. 1. Examples of 1984 tropical cyclones classified by track type. Straight-mover, TY Agnes (dashed line); recurver, ST Vanessa (dotted line); and odd-mover, ST Bill (solid line).

that the inclusion of synoptic predictors improved the model forecast skill, but none of his statistical models out-performed climatological forecasts. Lage (1982) used an empirical orthogonal function (EOF) representation of 500 mb geopotential height fields plus persistence-related variables to predict western North Pacific tropical cyclone recurvature or non-recurvature at 36-, 54- and 72-h forecast intervals. The combination of persistence plus EOF predictors consistently out-performed the persistence alone or the EOF predictors only methods. Each of these three techniques was superior to climatology and chance at all forecast times.

The purpose of this study is to test the feasibility of using an EOF representation of the synoptic vorticity fields at 700, 400 and 250 mb to identify recurvature situations in western North Pacific tropical cyclones. Because horizontal pressure gradients are generally weak and geostrophic relationships deteriorate in the tropics, geopotential heights provide a poor estimate of the steering flow. Since vorticity combines the steering effects of both zonal and meridional winds, it should provide a more accurate measure of steering with fewer predictors than would be required if the two components of the wind were used separately as predictors. An EOF representation of a synoptic

field such as vorticity offers several important advantages over gridpoint values. Because EOF predictors represent spatial patterns in environmental fields, they contain more synoptic information and are less affected by observational errors. Because relatively few EOF predictors are required to represent large amounts of variance in synoptic patterns, considerable savings in computer storage and forecast model run times can be realized using this method.

EOF predictors have been used successfully in statistical-synoptic models to forecast tropical cyclone motion (Shaffer and Elsberry 1982; Peak et al. 1986; Schott et al. 1987; and Elsberry et al. 1988). Shaffer (1982) demonstrated the usefulness of EOF representation of 500 mb geopotential heights as synoptic forcing predictors in statistical-synoptic track prediction schemes. In a similar study, Wilson (1984) used EOF representation of 700, 400 and 250 mb wind component fields to forecast tropical cyclone motion. Schott (1985) stratified forecast situations by the cyclone direction of motion to develop a statistical adjustment scheme involving EOF predictors that reduced the systematic errors in a dynamical track prediction model. Meanor (1987) used Schott's stratification scheme and EOF predictors of vertical wind shear to develop a similar model to adjust for systematic errors in a dynamical track prediction model. Weniger (1987) adopted Meanor's EOF predictors of vertical wind shear to develop a successful tropical cyclone intensity forecast model. Gunzelman (1990) used the EOF approach as a filter to represent the "signal" in the vorticity field, and suggested that several different forecast situations could be interpreted as an advection of these filtered vorticity fields.

B. OBJECTIVE

The objective of this study is to demonstrate the ability of an EOF representation of the synoptic vorticity field to identify potential recurvature situations. The hypothesis is that the adjacent synoptic features cause the turning motion that leads to tropical cyclone recurvature. Consequently, the sets of EOF coefficients for the vorticity fields associated with recurvature should be different from those associated with straight-track situations. The question is, how far in advance of recurvature are the recurvature EOF coefficients distinguishable from the straight-track EOF coefficients? Classification goals are two-fold: first, to identify the overall track type as a recurver versus a straight-mover; and second, to identify the time to recurvature with the best possible time resolution. Recurvature is defined here as the time when the storm heading changes from west of 000° North to east of 000° North. A track segment will be classified as a

straight-mover if the storm does not recurve during the next 72 h, which corresponds to the official JTWC forecast period. The time to recurvature will be specified in 12-h increments. In summary, the first goal of the study is to determine whether the present vorticity field is representative of a recurvature situation within 72 h versus that of a straight-mover; if so, the second goal is to specify the most likely time to recurvature.

Two methods are used to develop the classification model. In the Euclidean distance approach, classifications are into the group that has the closest mean EOF predictor values as measured in multidimensional space. This simple approach provides physical insight into the classification problem. The difficulty is in determining which predictors best separate the groups. Therefore, a discriminant analysis package also is used to more objectively demonstrate the predictive capabilities of an EOF representation of the vorticity field.

II. DATA AND METHODS

A. DATA DESCRIPTION

The cases in this study are 12-hourly data for western North Pacific tropical cyclones during 1979-1984. These cases are a combination of the cases analyzed by Wilson (1984), Peak et al. (1986) and Gunzelman (1990). Wilson and Peak et al. extracted the Global Band Analyses (GBA) wind fields for each case. Gunzelman computed the relative vorticity from these wind fields and performed the EOF analyses of the vorticity fields. The following restrictions were applied to the selection of cases:

- a tropical cyclone attaining at least tropical storm strength (maximum sustained winds of 18 m/s (35 kts) or greater);
- a best track position west of the dateline, east of 100° E and south of 34.6° N; and
- the meridional and zonal wind components of the GBA are available at 700, 400 and 250 mb.

A total of 1573 cases met these requirements and were analyzed.

1. Field description

The GBA wind fields are operationally generated every 12 h by the United States Navy Fleet Numerical Oceanography Center (FNOC). The GBA provide global longitudinal coverage between 40.956° S and 59.745° N. The analyses are produced on a Mercator grid with spacing of 2.5° latitude at 22.5° N and S. Although zonal and meridional wind fields also are available at the surface and 200 mb, only the 700, 400 and 250 mb levels are used. Analyses are based on surface observations, ship reports, rawinsondes, pibals, aircraft reports and satellite-derived cloud motion vectors. When a tropical cyclone is present, eight bogus winds are inserted at the surface 80 km (43 n mi) from the center of the cyclone, and are coupled vertically via the thermal wind equation using temperature analyses at the intermediate levels. A detailed description of the GBA is contained in the U.S. Naval Weather Service (1975).

Wilson (1984) and Peak et al. (1986) used a bi-linear interpolation scheme to interpolate the zonal (u) and meridional (v) GBA wind components onto a storm-centered grid for each case. The grid consists of 527 data points with a fixed zonal and meridional separation of 277.8 km (150 n mi). It is geographically-oriented with 17 gridpoints north-to-south and 31 gridpoints east-to-west. The center of the cyclone,

based on the JTWC warning position, is always located at gridpoint (16,9). Gunzelman (1990) computed relative vorticity

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \tag{2.1}$$

using centered finite differences at the internal gridpoints, and one-sided differences at the grid boundaries. Gunzelman noted that the mean vorticity fields are nearly vertical near the tropical cyclone, and the largest positive vorticity values around the cyclone are at 700 mb and decrease with height. The 700 mb vorticity field also has the largest difference between the positive values associated with the cyclone and the negative values associated with the subtropical ridge, and the gradient decreases with height. However, the magnitude of the vorticity associated with the subtropical ridge increases with height and is greatest at 250 mb.

2. **Empirical orthogonal function analysis**

The EOF method used by Gunzelman (1990) paralleled the procedures used by Wilson (1984) and Meanor (1987), except that it was applied to relative vorticity rather than wind components or the vertical wind shear. The EOF analysis was on the 527 point storm-centered grid (Section II.A.1) and was based on the same 682 dependent cases during 1979-1983 that Wilson used.

In this method, orthogonal eigenvectors and their associated eigenvalues (coefficients) are calculated from the dependent set vorticity fields at each pressure level. First, X (527) eigenvectors are calculated from a normalized XY (527 x 682) matrix of X (527) gridpoint values for the Y (682) cases. The original synoptic gridpoint values can be recovered by the linear summation of the products of the eigenvectors and their associated coefficients. The first eigenvector (spatial pattern) contains the largest variance. The second eigenvector contains the largest amount of the variance not explained by the first, and so on. Once the eigenvectors are determined from the dependent data set, the time dependence in the synoptic pattern for each case is contained in the EOF coefficients.

One of the advantages of the EOF representation is that a relatively small number of EOF eigenvectors can be used to represent a synoptic pattern. To determine the minimum number of eigenvectors that are needed to represent the signal in the vorticity field, Gunzelman (1990) applied the Preisendorfer and Barnett (1977) Monte Carlo technique to distinguish between eigenvectors with signal vice those with noise. In this method, eigenvalues for the physical data are compared to eigenvalues for

randomly generated data. If the physical eigenvalue deviates significantly from the eigenvalue computed from a random vorticity field, there is reasonable assurance that the associated eigenvector is describing signal rather than noise. Based on Gunzelman's (1990) results, the first 45 vorticity modes are retained as potential descriptors of the synoptic fields associated with recurvature in this study. The first 45 modes explain between 72.8 and 77.5% of the vorticities at the three pressure levels (Table 1).

Table 1. PERCENTAGE OF EXPLAINED VARIANCE WITH 1 TO 45 MODES: Cumulative percentage of variance (95% confidence) with 1 to 45 EOF modes retained for the relative vorticity fields at three pressure levels (after Gunzelman 1990).

MODE	700 MB	400 MB	250 MB
1	7.6	10.2	11.4
2	13.4	18.1	19.1
3	16.9	22.0	23.6
4	20.0	25.5	27.6
5	22.7	28.7	31.0
6	25.3	31.5	33.9
7	27.7	34.1	36.6
8	30.1	36.5	39.2
9	32.3	38.7	41.4
10	34.4	40.8	43.4
*	*	*	*
40	69.4	73.8	74.7
41	70.1	74.5	75.3
42	70.8	75.1	75.9
43	71.5	75.7	76.4
44	72.1	76.3	77.0
45	72.8	76.9	77.5

Each eigenvector consists of 527 values that represent a spatial pattern on the 31 x 17 analysis grid. The magnitude of the associated time-dependent EOF coefficient indicates the relative importance of that pattern in each specific case. A negative EOF coefficient indicates that the identical spatial pattern applies, except that the maxima and minima are reversed.

The first eigenvector for 700 mb vorticity (Fig. 2) can be interpreted as a tropical cyclone in the subtropical ridge if this pattern is multiplied by a negative coefficient. For example, the 700 mb Mode 1 coefficient for ST Vanessa at recurvature time is -4.57. Therefore, the opposite pattern with a positive vorticity value at the storm center (dot) applies, and represents a recurving tropical cyclone at the axis of the subtropical ridge. Mode 1 eigenvectors for 400 and 250 mb relative vorticity (not shown) represent

large-scale patterns similar to the Mode 1 pattern for 700 mb in Fig. 2. As the spatial patterns become increasingly more complex for higher mode eigenvectors, the patterns become increasingly more dissimilar among the three pressure levels (see Gunzelman 1990 for further discussion).

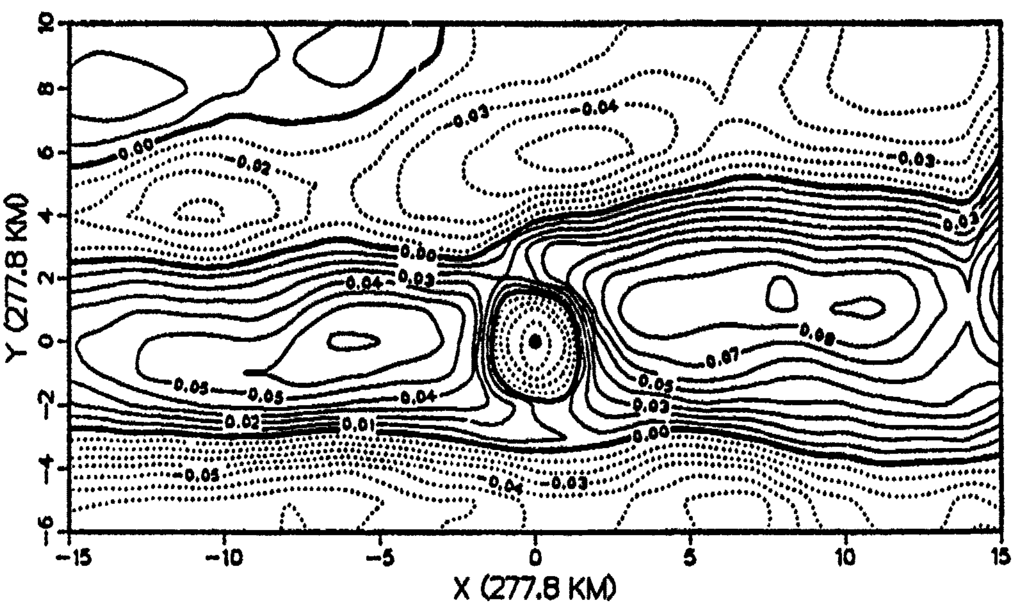

Fig. 2. Mode 1 eigenvector at 700 mb. Positive (negative) values are solid (dashed). North latitude is along the y-axis and east longitude is along the x-axis. The black dot indicates the storm center position (after Gunzelman 1990).

Reconstructed 700 mb vorticity fields for ST Vanessa at recurvature time using only the first 45 EOF modes and all 527 modes are compared in Fig. 3. The basic pattern of a tropical cyclone at the axis of the subtropical ridge with a strong vorticity gradient to the east, and cyclonic vorticity associated with the midlatitude trough to the north, is represented equally well with 45 EOF modes as with all 527 modes. The addition of the higher EOF modes adds smaller scale features, which are assumed to represent noise in the vorticity field.

B. SELECTION OF CASES

A recurvature forecast model learning set is selected from the 1573 cases in the 1979-1984 data set. As a first step in the selection process, the data are categorized by track type and time to recurvature. Initial identification of the cases as recurvers, straight-movers and odd-movers is based on the tropical cyclone track categories

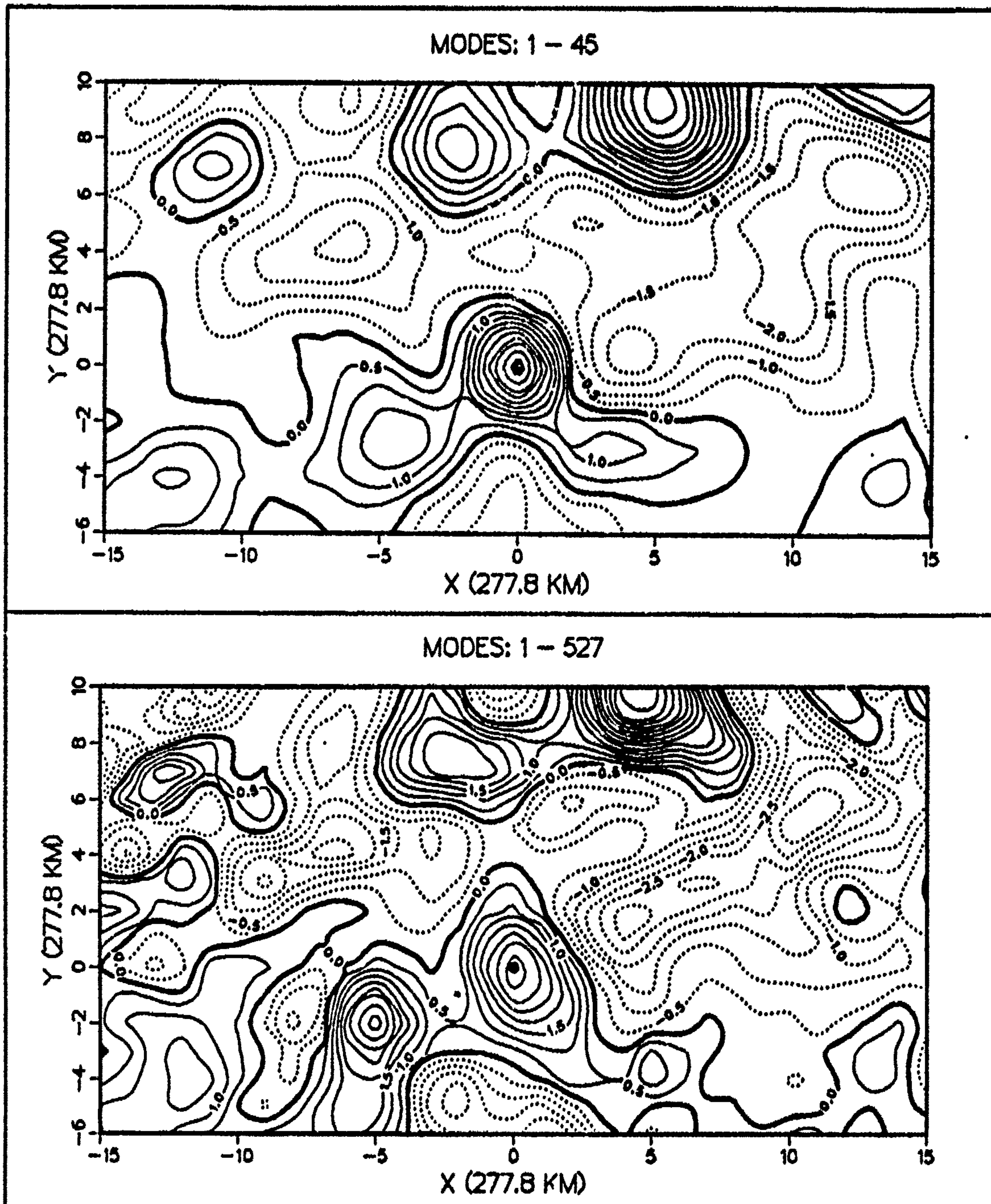

Fig. 3. Reconstructed 700 mb vorticity for ST Vanessa at recurvature. Relative vorticity contours ($10^{-5}s^{-1}$) reconstructed from the first 45 EOF modes (top) and from all 527 modes (bottom). Positive (negative) values are solid (dashed). North latitude is along the y-axis and east longitude is along the x-axis. The black dot indicates the storm center position.

assigned by Miller et al. (1988). For each recurving tropical cyclone, the storm heading between successive 6-h JTWC best track positions is computed and the recurvature time is identified as the 00 or 12 UTC nearest the 6-h interval in which the storm heading changed from west of 000° North to east of 000° North. This synoptic map time, for which a GBA is available to calculate the vorticity EOF coefficients, will be referred to as R-00h where R indicates recurvature and -00h indicates the number of hours (0) prior to recurvature time. Recurver cases within 96 h of recurvature are then categorized based on the time to recurvature into the R-96h through R-00h classification groups. Cases more than 96 h prior to recurvature are identified as pre-recurvers (PR). Cases after recurvature are excluded from the forecast model learning set. The straight-mover cases are identified as non-recurvers (NR) if a minimum of 72 h remains in the track to establish that recurvature does not follow in that time. This requirement excludes from the learning set all straight-mover cases that cannot be verified as non-recurvature situations throughout a 72-h forecast period. Odd-mover cases (382 cases from 33 tropical cyclones) are not included in the model learning set, but will be used to test the ability of the final EOF recurvature forecast model to classify these cases into the straight-mover or recurver group that most closely describes the storm motion.

After screening, a total of 782 cases from 97 storms are retained in the model learning set (Table 2). Although the learning set cases in the Euclidean distance approach and the discriminant analysis approach differ, the entire learning set will be used to compare the overall prediction skill of the approaches.

C. CRITERIA FOR EVALUATING MODEL PERFORMANCE

Evaluation criteria are chosen to test the forecast model's ability to meet the two classification goals: identification of track type and identification of the time to recurvature. Since no objective guidance is available (or official forecast is issued) as to whether a storm will be a recurver or a straight-mover, the only absolute measure of usefulness is a comparison with a climatological forecast of recurvature.

1. Percent correct

The percent of cases correctly forecast as recurver (%R) or straight (%S) and the total correctly forecast in both track type categories (%T) tests the model's ability to identify the overall track type. The percent correct is calculated for recurver and straight-track types defined by the times in Table 2. That is, a classification into any of the R-72h through R-00h groups is considered to be a correct forecast of a recurver. Similarly, classification into the NR, PR and R-96h through R-84h groups represent

Table 2. RECURVATURE MODEL LEARNING SET CASES BY FORECAST CATEGORY: Number of 1979-1984 tropical cyclones that are categorized as recurver or straight track types. The recurver learning set is defined as those times within 72 h of recurvature time (R-00h). The straight learning set includes all times preceding 72 h of recurvature time, plus selected times from the straight-track storms. The number of cases retained in the model learning set is listed for each track-type category and for each 12-h forecast category.

TRACK TYPE	NUMBER OF TROPICAL CYCLONES	12-H FORECAST CATEGORIES	NUMBER OF CASES
RECURVER	60	R-00H	55
		R-12H	56
		R-24H	55
		R-36H	52
		R-48H	46
		R-60H	41
		R-72H	32
		TOTAL	337
STRAIGHT	37	R-84H	30
		R-96H	24
		PR	113
		NR	278
		TOTAL	445
TOTAL	97		782

correct forecasts of a straight-track situation, because the tropical cyclone did not recurve during the 72-h forecast period. The simple percent correct measure is also used in evaluating the time-to-recurvature prediction performance of the model. In that case, only a classification into the appropriate time-to-recurvature group will be credited as a correct forecast.

2. Classification matrix scores

Classification matrix scores assign penalty points to misclassifications as a linear function of the number of 12-h categories between the prediction and the verification groups. That is, one additional penalty point is assigned for each 12-h group between the model forecast and the verification. Since a misclassification of a recurvature case into the PRNR forecast group represents a larger error, two additional penalty points are assigned in the PRNR category relative to the R-96h category. Because this is a

penalty score, higher skill is represented by numbers close to zero. A penalty score of 1.0 would indicate that the average misclassification is off by one category.

Three classification matrix scores are defined based on the matrix of penalty points in Table 3: D-score (dependent); I-score (independent); and R-score (recurver). Given a classification matrix that contains the number of cases that are forecast in each classification group (columns) and verify in each verification category (rows), the penalty points in Table 3 are assessed by multiplying the number of cases by the penalty points for that error. No penalty points are given to the correct classifications along the diagonal.

Table 3. MATRIX OF PENALTY POINTS FOR CLASSIFICATION MATRIX SCORES: Penalty points are assessed for erroneous forecasts of time-to-recurvature in 12-h increments or as PRNR. These penalty points are summed over three subsets to calculate the classification matrix D-, I- and R-scores. The matrix columns (forecast model classification groups) and rows (case verification categories) are the same as those in the model classification matrix.

	CLASSIFICATION									
VERIFY	00	12	24	36	48	60	72	84	96	PRNR
R-00H	0	1	2	3	4	5	6	7	8	10
R-12H	1	0	1	2	3	4	5	6	7	9
R-24H	2	1	0	1	2	3	4	5	6	8
R-36H	3	2	1	0	1	2	3	4	5	7
R-48H	4	3	2	1	0	1	2	3	4	6
R-60H	5	4	3	2	1	0	1	2	3	5
R-72H	6	5	4	3	2	1	0	1	2	4
R-84H	7	6	5	4	3	2	1	0	1	3
R-96H	8	7	6	5	4	3	2	1	0	2
PR	10	9	8	7	6	5	4	3	2	0
NR	10	9	8	7	6	5	4	3	2	0

The three classification matrix scores are obtained by multiplying the classification matrix of model results by the penalty point matrix and calculating three sums of the products. These sums are then normalized by the number of cases in the sample so that the scores can be compared for different sample sizes. The three classification matrix scores examine various aspects of the forecast model skill by scoring only cases that belong to certain verification categories. The classification matrix I-score includes cases in all of the verification categories (R-00h through R-96h plus PR and NR) that are in the independent sample. The D-score is designed to compare results from dependent

and independent samples, which contain different sets of cases. Since the PR cases are not always included in the dependent set to define a PRNR classification group, the PR case forecasts are excluded in the classification matrix D-score. Consequently, the D-score and I-score will have similar magnitudes, with an offset that is proportional to the performance of the forecast model on the PR cases. The D- and I-score will provide an exact comparison of forecast skill only if the ratio of the combined number of PR and NR cases to the combined number of R-00h through R-96h cases is the same for both data sets (e.g., in the learning set). Since the PR and NR cases are assigned more penalty points for misclassifications than the R-00h through R-96 cases, the relative number of cases in each group will affect the matrix scores that score PR and NR forecasts. The R-score is an indication of the model's ability to correctly identify the time to recurvature in recurver cases. That is, the penalty scores in Table 3 are only summed over the R-00h through R-72h verification categories.

3. Climatological forecasts and scores

A climatological forecast is obtained by counting the number (N in Table 4) of JTWC best track 00 and 12 UTC positions for 1979-1984 cyclones of tropical storm strength or greater in each classification group (R-00h through R-96h plus PRNR). Thus, the climatology data set contains all the learning set cases, plus additional cases that were excluded from the learning set because either the best track position did not meet the requirements in Section II.A or the GBA wind fields were not available at all three pressure levels. The percentage of recurving (41.7), straight-moving (36.4) and odd-moving storms (21.9) for these six years is representative of the percentages (42.5, 36.4 and 21.1, respectively) for the 28-year period 1945 to 1987 (Miller et al. 1988).

To obtain the climatological forecast classification matrix (Table 4), a fraction of the learning set cases in each 12-h verification category are forecast into each of the ten classification groups based on the percent of climatological cases in each of the ten groups (percent in Table 4). By ignoring the straight-mover cases with less than 72 h remaining and the odd-mover cases, these climatological forecasts can be compared to the model forecasts predicated on similarly screened data. The skill scores for the climatological forecasts of the learning set cases are given in Table 5. Any forecast model should have higher percent correct and lower D-score, I-score and R-score to be considered as useful to the forecaster.

Table 4. **CLIMATOLOGICAL FORECASTS FOR THE LEARNING SET:** The learning set cases belonging to each verification category are classified into the ten classification groups with the relative frequency (column labeled percent) that the cases in the 1979-1984 climatology data set belong to each of the ten classification groups. Since the number of classifications is rounded to the nearest whole integer, the total is 780 vice 782.

		CLIMATOLOGY		LEARNING SET CLASSIFICATIONS									
	VERIFY	N	PERCENT	00	12	24	36	48	60	72	84	96	PRNR
RECURVER:	R-00H	61	(6.31)	3	3	3	3	3	2	2	2	1	32
	R-12H	60	(6.21)	4	3	3	3	3	2	2	2	1	32
	R-24H	58	(6.00)	3	3	3	3	3	2	2	2	1	32
	R-36H	53	(5.49)	3	3	3	3	2	2	2	2	1	30
	R-48H	46	(4.76)	3	3	3	3	2	2	2	1	1	27
	R-60H	42	(4.35)	3	3	2	2	2	2	1	1	1	24
	R-72H	33	(3.42)	2	2	2	2	2	1	1	1	1	19
STRAIGHT:	R-84H	30	(3.11)	2	2	2	2	1	1	1	1	1	17
	R-96H	24	(2.48)	2	1	1	1	1	1	1	1	1	14
	PR	114	(PRNR=	7	7	7	6	5	5	4	4	3	65
	NR	445	57.87)	18	17	17	15	13	12	9	9	7	161

Table 5. **FORECAST SKILL FOR CLIMATOLOGICAL FORECASTS:** Percent of recurver (%R), straight (%S) and total (%T) cases correctly classified according to track type. D-, I- and R-score are classification matrix scores that indicate skill in correctly classifying cases with 12-h accuracy. Scores are computed from the actual number of learning set cases that climatologically occur in each group vice the integer values presented in Table 4.

%R	36.5
%S	63.5
%T	53.6
D-score	4.11
I-score	3.93
R-score	5.30

III. EUCLIDEAN DISTANCE METHOD

A. BACKGROUND

The Euclidean distance approach in this section examines both the physical changes in the vorticity patterns that precede tropical cyclone recurvature and the ability to distinguish among these patterns using an EOF representation. Since the time-dependent EOF coefficients represent the synoptic fields that exist at each time, the coefficients should vary in a systematic manner as the tropical cyclone moves around the subtropical ridge during recurvature. Simple two-dimensional plots of the first and second EOF coefficients on the x- and y- axes in Fig. 4 indicate that these coefficients for the 1984 recurvers have similar traces. The Mode 1 coefficients are initially positive, which indicates a large-scale positive vorticity pattern centered along the latitude of the storm center in the first eigenvector (Fig. 2) and represents the synoptic pattern while these storms are still located in the monsoon trough. As these storms move northward out of the monsoon trough and recurve, the magnitude of the Mode 1 coefficients decreases and then becomes negative to represent the negative vorticity associated with the subtropical ridge. At the time of recurvature, the first and second EOF coefficients for the 1984 recurvers tend to cluster in the same region on the two-dimensional plot. In contrast, the 1984 straight-moving cyclones have EOF coefficients that cluster in a separate region, and the odd-moving cyclones have coefficients that exhibit characteristics of both the recurvers and straight-movers (Fig. 5). This leads to the hypothesis that an individual cyclone may be distinguished as a recurver (straight-mover) if the EOF coefficients for that cyclone are closer to the mean of the cluster associated with recurvers (straight-movers). The questions are how far in advance of recurvature can these differences in EOF coefficients be detected and with what time accuracy.

B. MODEL DEVELOPMENT

To test the hypothesis that individual cases may be classified according to the closeness of their EOF coefficients to the mean values for the recurver and straight sets, a classification model is developed using the Euclidean distance method. The Euclidean distance (D) is calculated in multidimensional EOF space using the formula

$$D = \sqrt{(\alpha(a) - \overline{\alpha}(a))^2 + \ldots + (\alpha(i) - \overline{\alpha}(i))^2}\,, \tag{3.1}$$

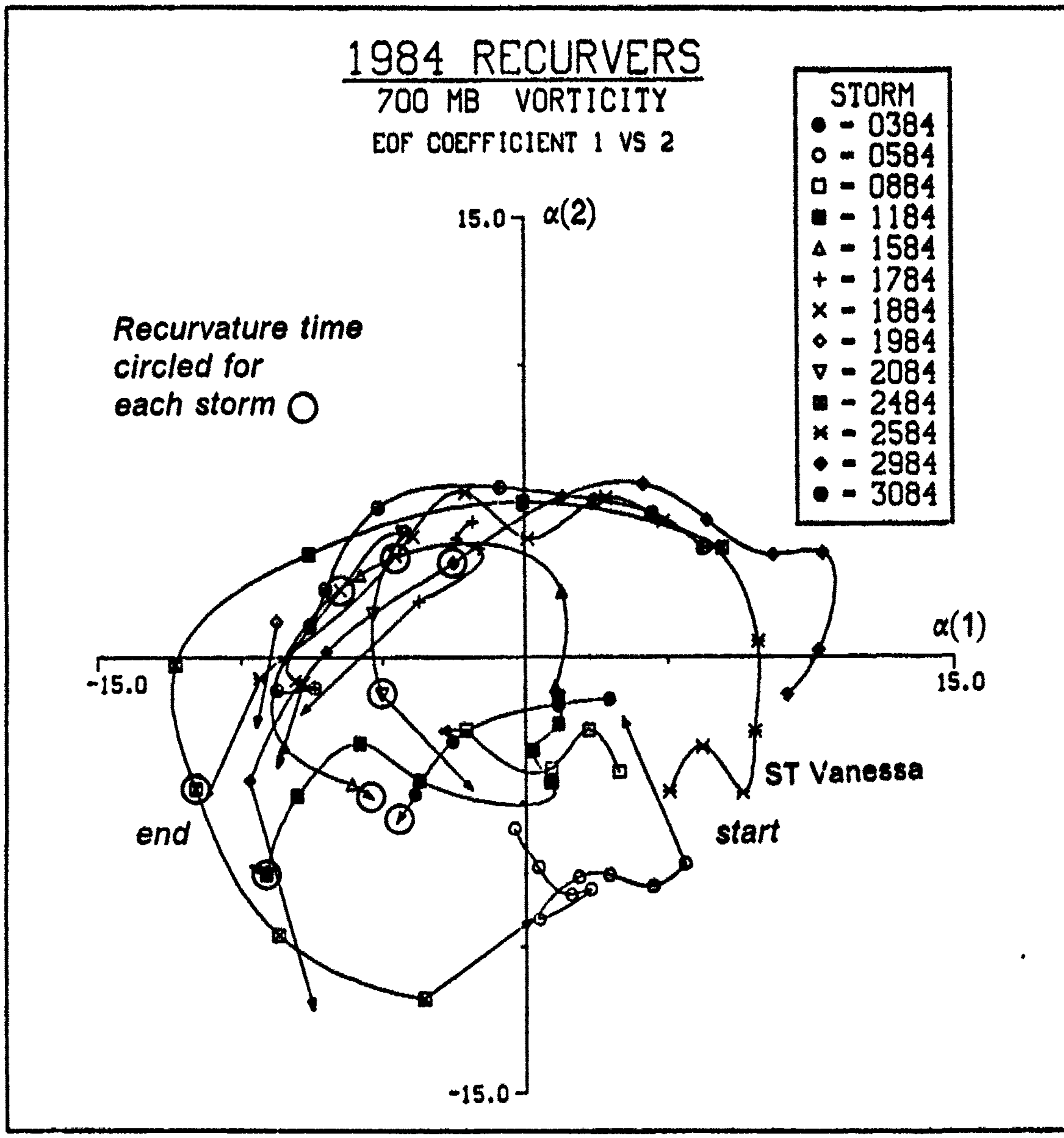

Fig. 4. Time progression of the first and second EOF coefficients for 1984 recurvers. Markers indicate the values of the first (x-axis) and second (y-axis) EOF coefficients of the 700 mb vorticity fields for all 12-hourly cases analyzed by Gunzelman (1990). Values at recurvature time are circled and arrow heads mark the last case in each storm sequence (see legend for storm number). The start and end of the sequence for ST Vanessa (storm number 25 during 1984 is denoted 2584) are labeled.

where α is the EOF coefficient for the case, $\bar{\alpha}$ is the mean of the EOF coefficients for the forecast classification group, and the indices a through i represent the EOF modes used as predictors. Separate distances are calculated relative to the mean EOF value of each

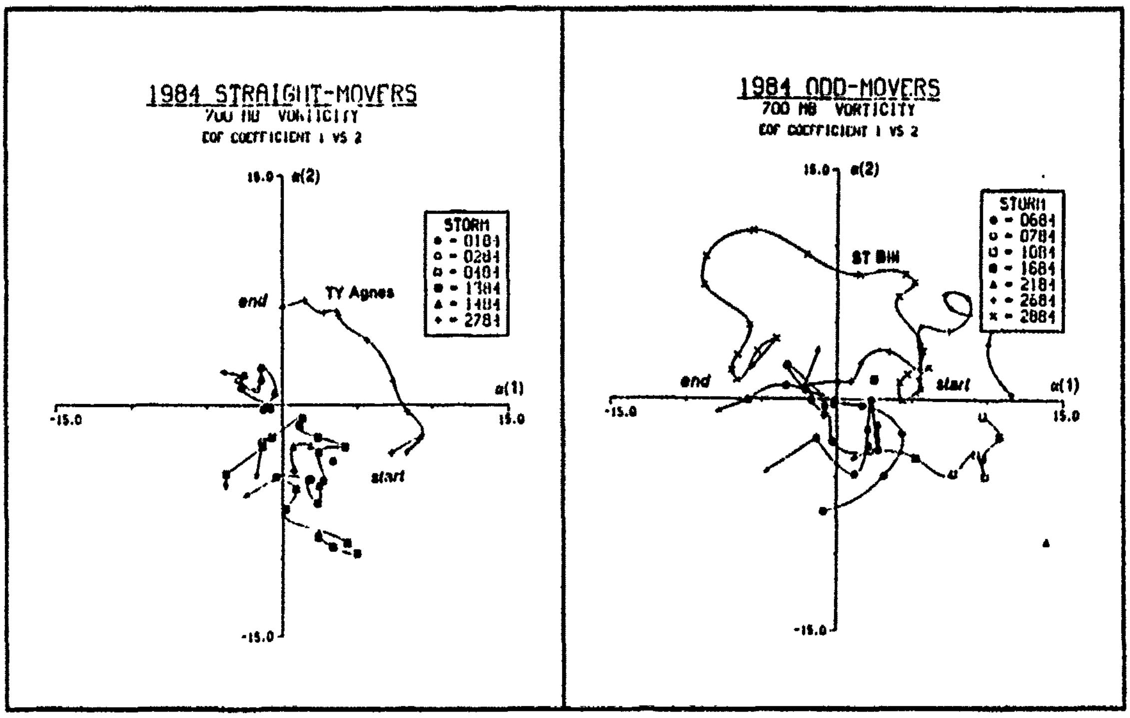

Fig. 5. Time progression of the first and second EOF coefficients for 1984 straight-movers and odd-movers. As in Fig. 4, except for 1984 straight-moving storms (left) and odd-moving storms (right).

potential classification group, and then the classification is into the group with the smallest distance.

1. Forecast group means

Two issues in the development of this simple model are the selection of the representative recurver and straight-mover cases to calculate the classification group means, and the specification of the set of EOF modes that best distinguishes between the recurver and straight-mover situations. A "clean" set of 15 recurving and 15 straight-moving storms is selected from the 1979-1984 data set in hopes of identifying the most representative vorticity patterns for the classification categories. The following criteria are used to select the clean sets:

- a tropical cyclone attaining a least typhoon strength (maximum sustained winds of 33 ms^{-1} (65 kts) or greater);
- formation east of 130° E; and
- a typical recurver or straight track exhibiting no significant deviations.

Tracks for the clean set storms are shown in Fig. 6. Because the clean storms exhibit typical recurver- or straight-track motion, the EOF coefficients for these storms should be representative of the typical vorticity patterns associated with recurver- or straight-track motion.

The mean values of the first 45 time-dependent EOF coefficients are computed from the clean recurver set at times R-00h through R-96h and from the clean straight-mover set that is labeled NR. As in the time progressions of the first two EOF coefficients for the 1984 storms in Figs. 4 and 5, considerable variability exists around the 12-hourly mean coefficients even in these clean set storms (not shown). To obtain more representative transitions among the time-to-recurvature groups in EOF space, a running mean value is calculated from three times centered on the desired time. For example, the mean for recurvature time R is calculated from the EOF coefficients at R-12h, R-00h and R+12h. The NR group averages also are calculated from three consecutive 12-hourly cases. These cases are selected so that the average longitude of the clean set straight-mover cases (132.06° E) is close to the average longitude of the clean set recurvers at recurvature time (130.99° E). Although only straight-mover data are used to define the PRNR classification group, the Euclidean distance approach should distinguish straight-moving cases (NR) as well as recurving storm cases that are more than 96 h before recurvature (PR).

Vorticity fields at each pressure level (700, 400 and 250 mb) reconstructed from the mean EOF coefficients for each classification group (Figs. 7, 8 and 9) illustrate the evolution of the synoptic patterns associated with recurvature. These patterns are similar at all three levels. The sequence starts with the NR pattern in which the subtropical ridge is well defined by the broad anticyclonic (negative) vorticity center to the north of the cyclone center. Such a pattern would be expected to produce westerly or northwesterly storm motion and a straight-type track. At R-96h, the anticyclonic vorticity associated with the subtropical ridge is weaker to the north and stronger to the northeast of the storm center than it was in the NR pattern. Proceeding toward recurvature time, the cyclonic (positive) vorticity associated with the storm and the anticyclonic vorticity associated with the subtropical ridge increase in magnitude as the composite "clean-set storm" moves north-northwest around the ridge. At recurvature time, the storm center position is at the axis of the ridge and only a relatively weak region of anticyclonic vorticity is found between the storm and the midlatitude cyclonic vorticity to the north. The differences among the recurvature patterns at the three

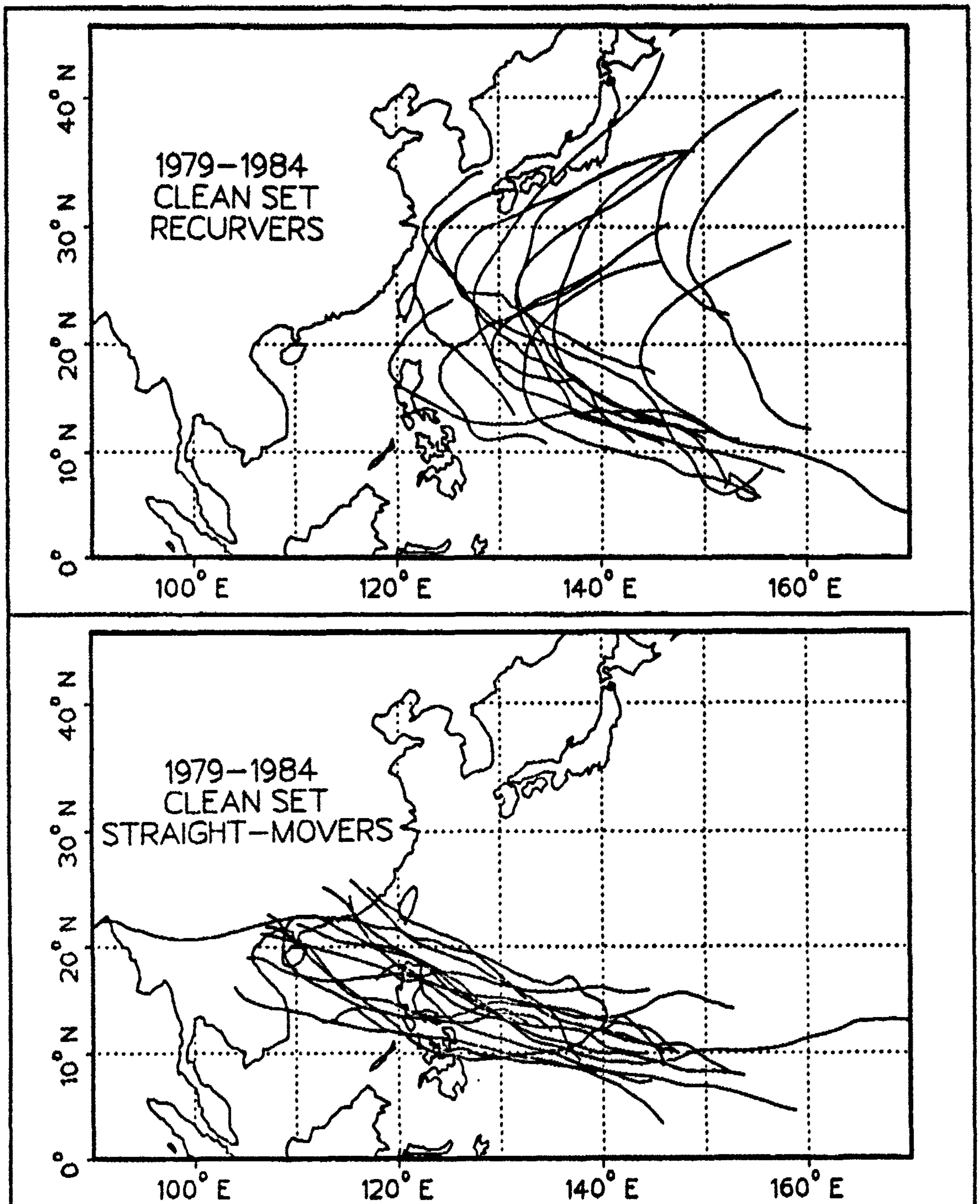

Fig. 6. Clean sets of recurvers and straight-movers for the Euclidean distance approach. JTWC best tracks for the 15 recurving (top) and 15 straight-moving (bottom) clean set storms during 1979-1984. These storms are used to calculate the mean time-dependent EOF coefficients that identify the recurvers and the straight-movers for the Euclidean approach.

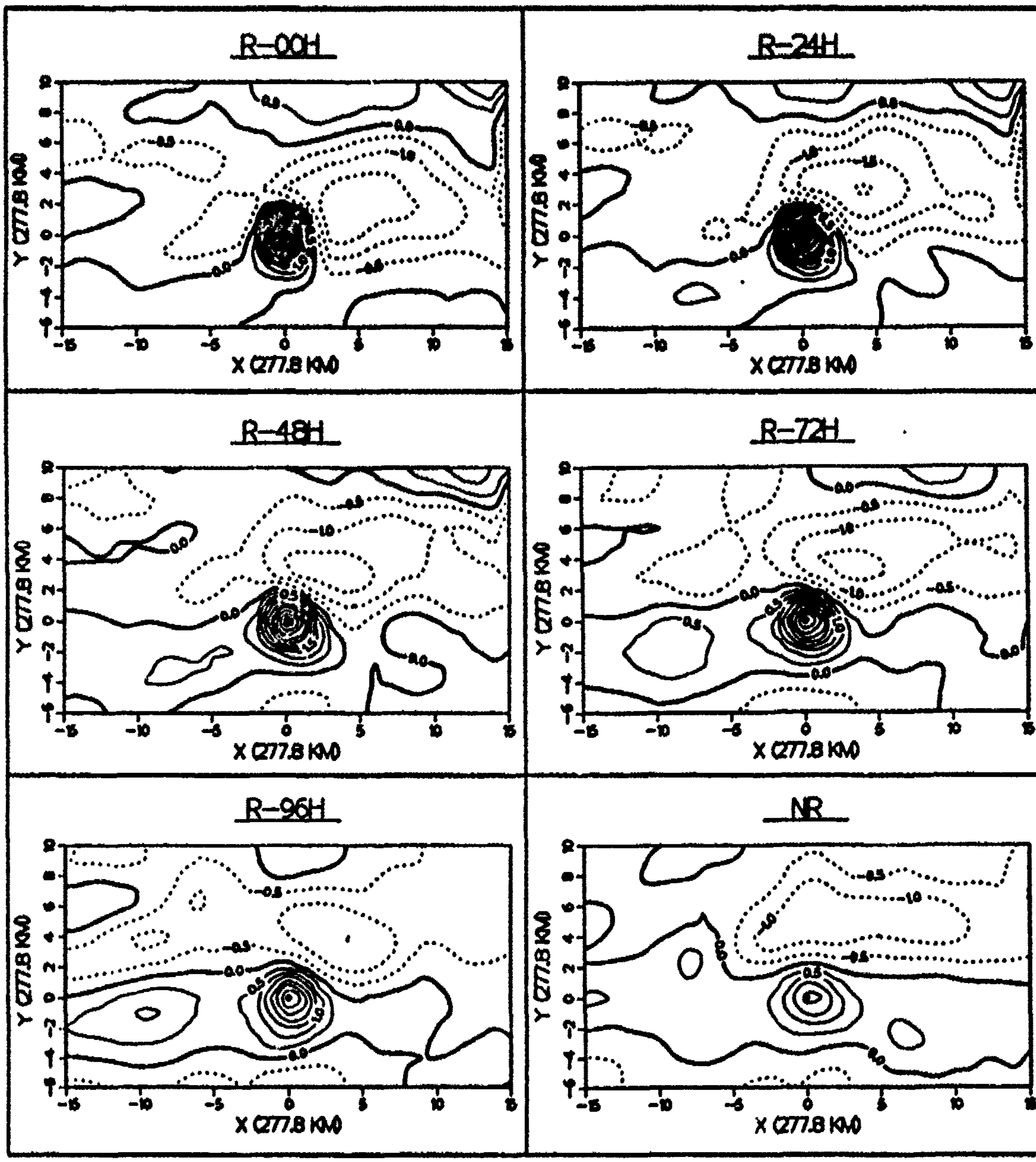

Fig. 7. Reconstructed 700 mb vorticity fields for clean set composites. Relative vorticity contours ($10^{-5}s^{-1}$) are reconstructed from the means of the first 45 EOF modes for the clean set storms at R-00h (top left), R-24h (top right), R-48h (middle left), R-72h (middle right), R-96h (bottom left), and NR (bottom right). Positive (negative) values are solid (dashed). North latitude is along the y-axis and east longitude is along the x-axis. The black dot indicates the storm center position.

pressure levels are similar to the relative vorticity differences with height noted by Gunzelman (1990), as described in Section II.A.1.

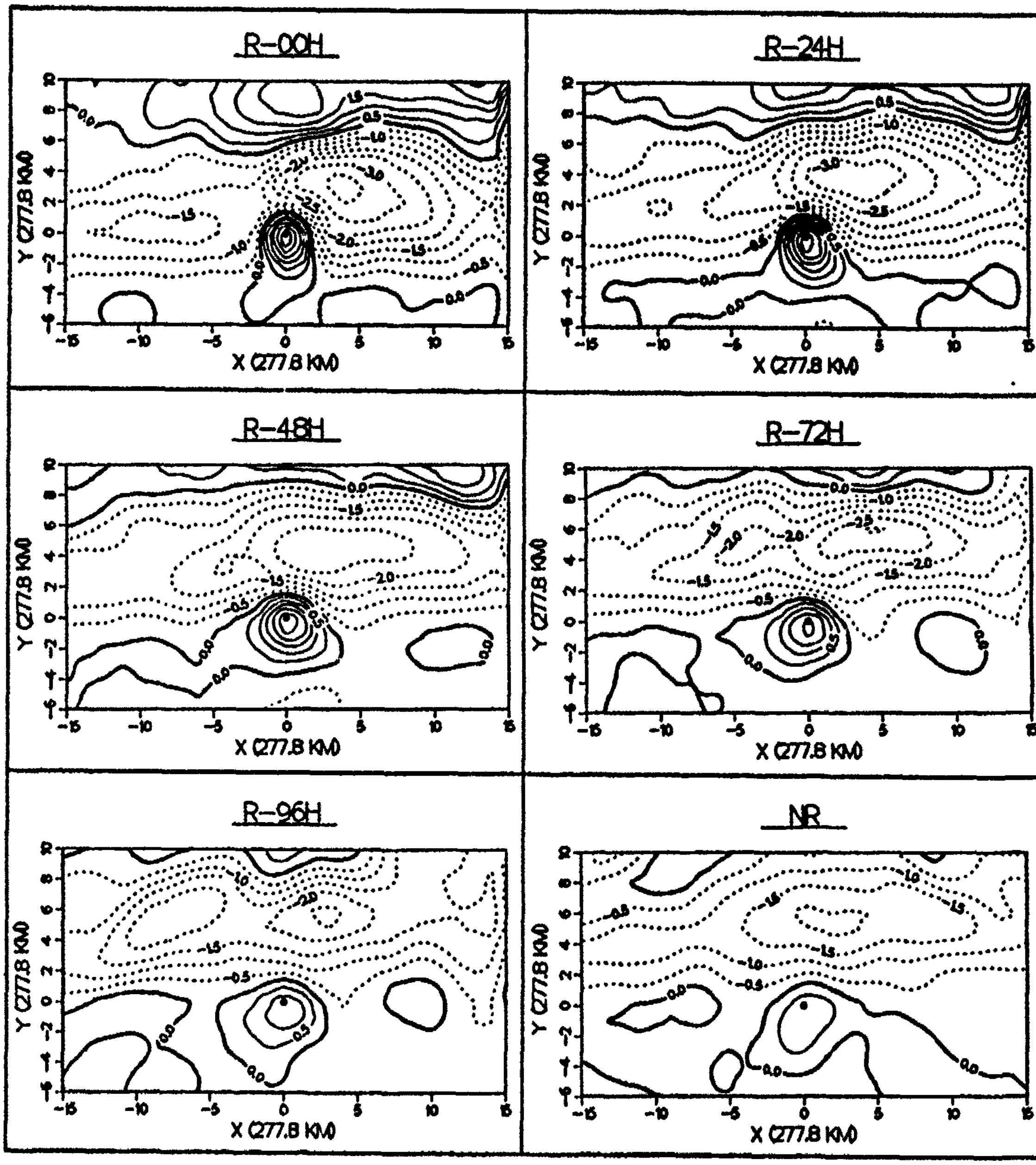

Fig. 8. Reconstructed 400 mb vorticity fields for clean set composites. Time intervals and contours are similar to Fig. 7.

2. Predictor modes

The objective is to select the set of EOF predictors that best separates the time-to-recurvature and PRNR classification groups, as defined by the clean set mean values in multidimensional space. Since the Euclidean distance approach offers no

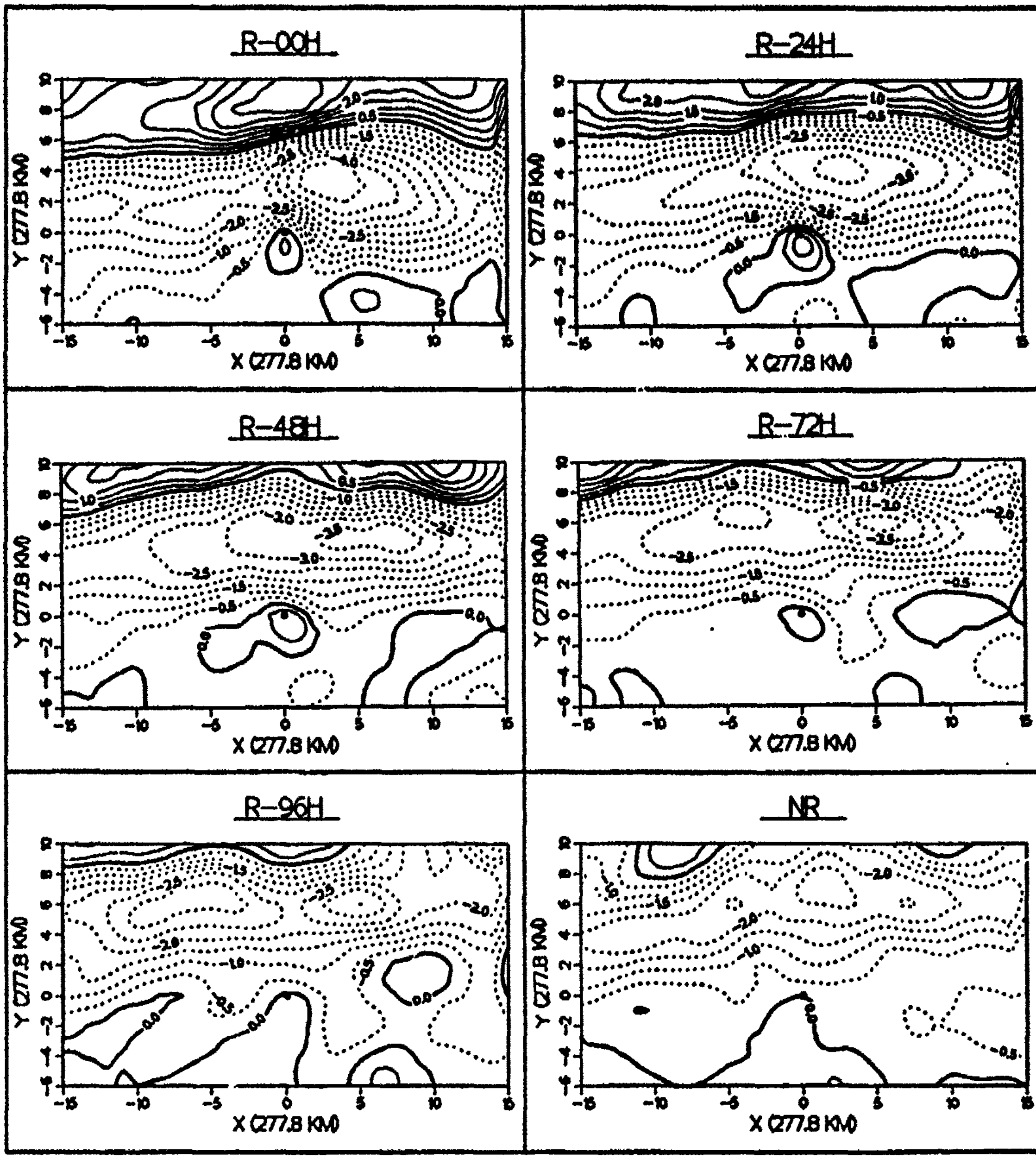

Fig. 9. Reconstructed 250 mb vorticity fields for clean set composites. Time intervals and contours are similar to Fig. 7.

objective selection criteria, such as the F-to-enter and other statistics in regression and discriminant analysis packages, the final choice of predictors will be based on model classification skill. Although the initial tests are conducted for all three pressure levels, the Euclidean model development is presented here for 700 mb data only.

Potential predictors are first screened by the ability to distinguish the clean set recurving cases from the straight-moving cases (NR) at each 12-h time (R-00h through R-96h). In this procedure, the clean set cases in one time-to-recurvature group (plus or minus 12 h) and all straight-moving storm cases are classified as recurvers (straight-movers) if the Euclidean distance is closer to the clean set mean values for the time-to-recurvature group (PRNR group). The skill in identifying the storm type is expressed as the percent correctly classified.

To illustrate the importance of the choice of the predictor set modes, clean set classifications into recurvers versus straight-movers using 200 randomly selected sets of ten EOF modes are compared in Fig. 10. One hundred sets of ten modes are selected randomly from the first 45 EOF modes (top) and 100 sets are formed from EOF Mode 1 plus nine other randomly selected modes (bottom). The combined skill in distinguishing between recurving and straight-moving storms is better than 50% for all times before recurvature for all random sets. The highest skill is achieved when EOF Mode 1 is forced and ranges from 95% at R-00h to about 80% at R-48h to R-96h. However, the skill among the random sets varies by as much as 40 percentage points. In the tests with ten randomly selected predictors (top, Fig. 10), notably better classification skill in the R-00h through R-36h groups also is achieved when Mode 1 is included. These results illustrate the importance of Mode 1 in distinguishing recurving storm vorticity fields near recurvature time. However, the remaining EOF predictors are necessary to discern the R-48h through R-96h recurving storm vorticity fields from the straight-mover fields. The problem is how to determine the optimum set of predictors without having to evaluate all possible permutations of the first 45 EOF modes.

Since the optimum set of predictors must be able to distinguish recurver and straight vorticity fields at all 12-h time steps before recurvature, the set should consist of some combination of the modes that best distinguish at each of the individual times before recurvature. Thus, recurver versus straight-mover classifications are evaluated for each of the 45 EOF modes separately for each 12-h time group. For each time-to-recurvature group, the first 45 modes are ranked as potential predictors in the order of their individual skill. Then a prototype set of predictors is formed from the two predictors with the highest individual skill. If the skill for this set is greater than when only the highest individual predictor is included, the second predictor is retained in the set. This stepwise process is continued by including the individual predictor with the next highest skill until the 45th best EOF mode is evaluated. In each step, the new predictor is only retained if the percent correct classifications is increased over the previous step.

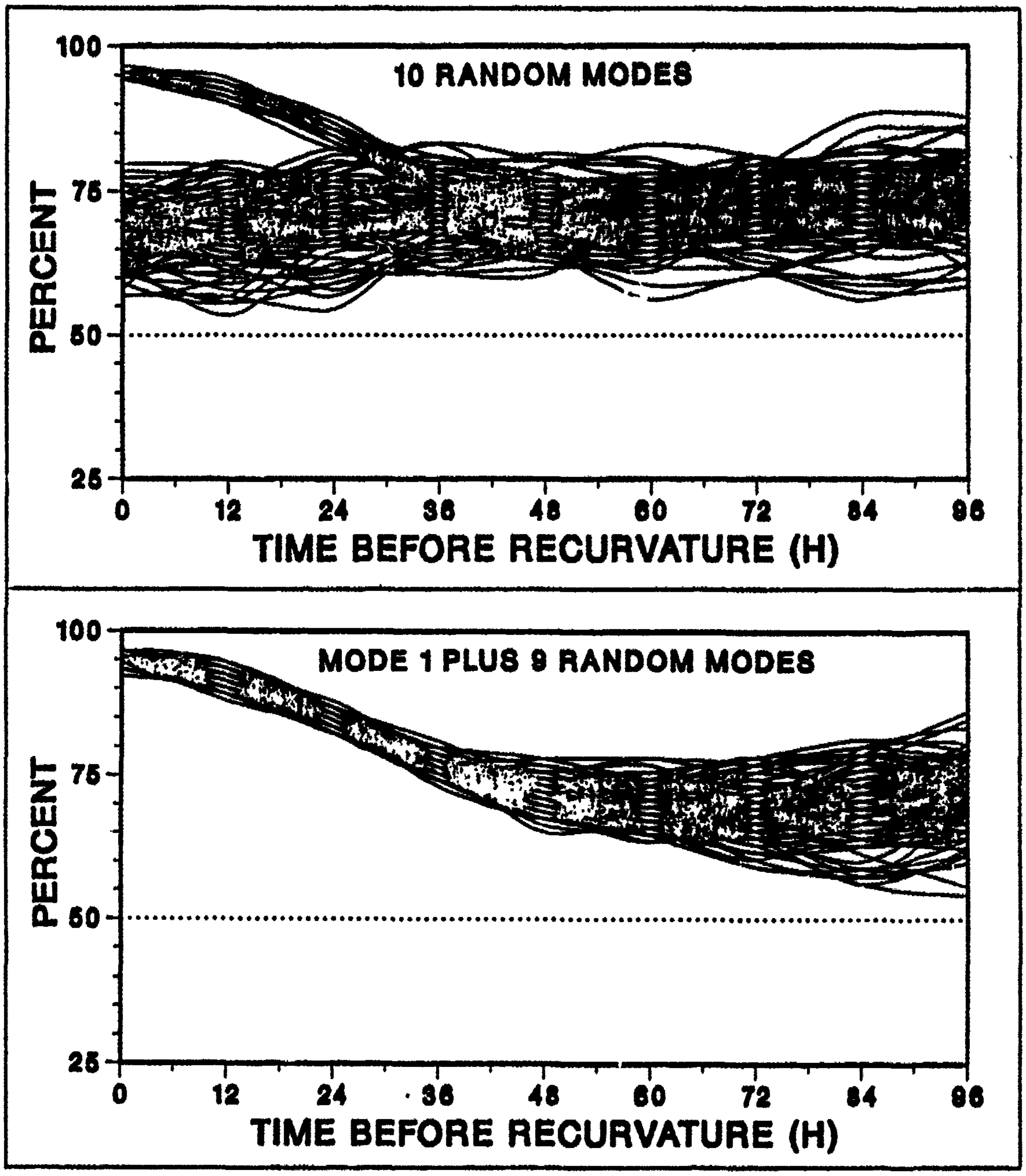

Fig. 10. Euclidean method classification skill into recurvers and straight-movers using randomly selected EOF predictors. Classification skill for the clean set cases using 100 sets of ten randomly selected EOF modes (top) and using 100 sets of EOF Mode 1 plus nine other randomly selected modes. Clean set cases in each time-to-recurvature group (R-00h through R-96h) (abscissa) are distinguished from the clean set straight-mover cases (NR). The percent correct classifications (ordinate) is for both the recurving and straight-moving storm cases.

Surprisingly, only the EOF Mode 1 is included in the 700 mb set for the R-00h group using this stepwise screening process. That is, no other mode increases the classification skill relative to EOF Mode 1 alone. However, the skill of this Mode 1 Euclidean model in distinguishing clean set recurver and straight-mover cases (top, Fig. 11) rapidly declines from 95% for R-00h to 75% for R-36h and only 50% at R-96h. This result is consistent with Fig. 10, and indicates that Mode 1 alone is not adequate for distinguishing recurvers versus straight-movers at other times prior to recurvature. When the stepwise addition of predictors is applied at each of these times, multiple modes are selected. The skill of these sets of predictors to distinguish recurver and straight storm cases (bottom, Fig. 11) ranges from 80-95%. Consequently, this result indicates the optimum performance of an Euclidean model with the dependent set of clean storms. In practice, the time to recurvature is unknown and the forecaster would not know which of these sets for individual times would apply. The objective is then to select a set of predictors that can be applied at all times, but does not degrade too severely from the optimum performance at the individual times shown in Fig. 11.

Potential overall best sets are formed from the EOF modes included in the separate sets determined for each time step in Fig. 11 plus other time-step sets. Two additional time-step sets are formed using the less restrictive selection criteria that inclusion of a specific EOF mode does not change (degrade) skill. In another time-step set selection approach, the EOF modes simply are entered in numerical order, rather than according to their relative skill in discerning storm type. Using the lower mode EOF coefficients, which are less likely to contain noise than the higher modes and are related to larger scale features in the vorticity fields, may provide more reliable separation among the classification groups. A summary of these five selection criteria for the time-step sets is given in Table 6. Since each of these selection criteria leads to the inclusion of different EOF modes in the Euclidean method for the time-step groups, no consensus is evident for use in forming the overall best sets.

Various subjective criteria involving the number of times an EOF mode is selected for one of the individual time-step sets are tested to form an overall best set. For the collection of R-00h through R-96h predictor sets selected using one of the criteria A through E in Table 6, a mode may be required to appear in a certain number of these individual sets to be included in a potential overall best set. Each potential overall best set of predictors is evaluated by scoring the Euclidean distance classifications into the 12-h time-to-recurvature groups (R-00h through R-96h) plus PRNR. Classification

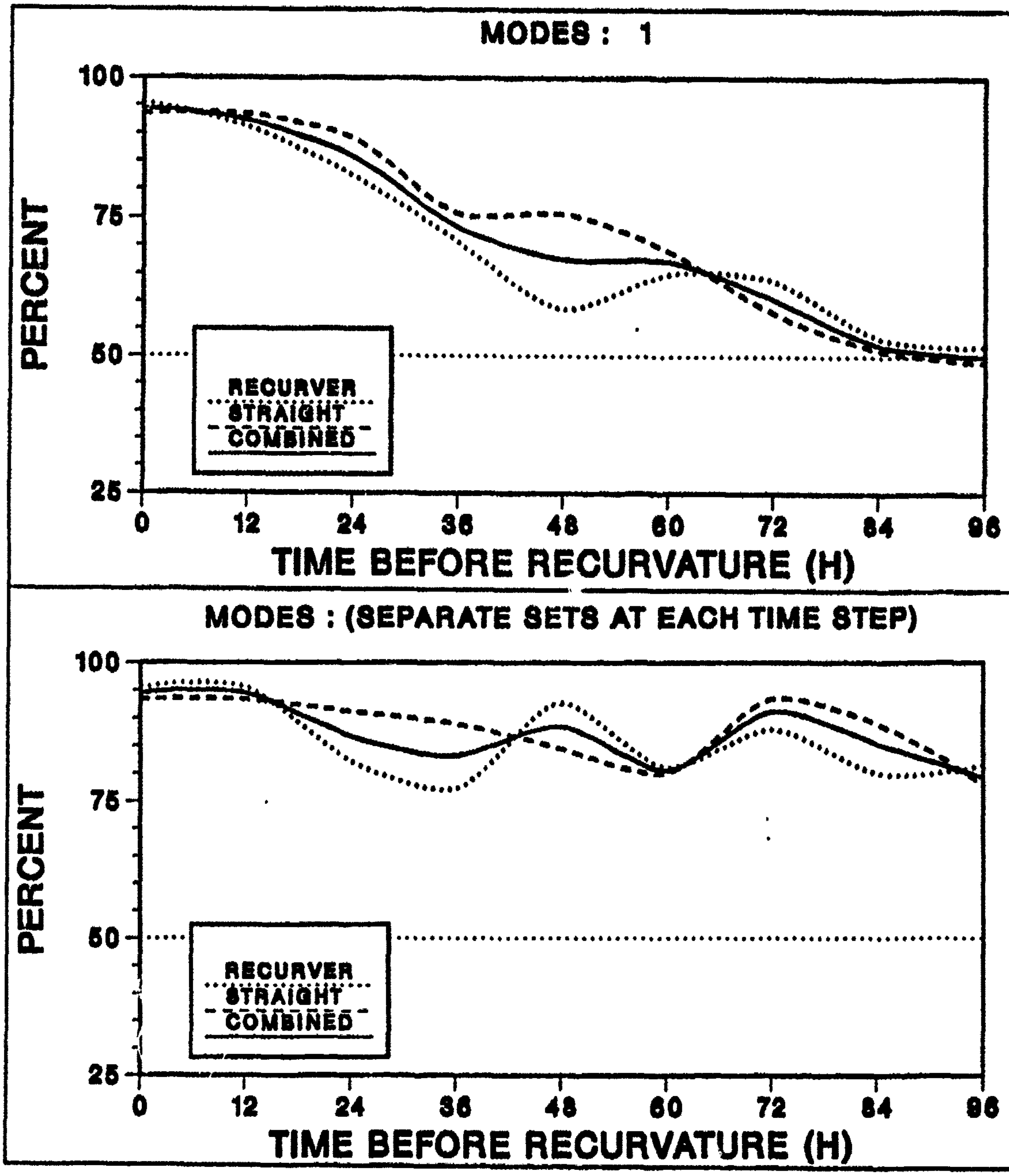

Fig. 11. Euclidean method classification skill using time-step sets of EOF predictors. Classification skill as in Fig. 10, except for only the EOF Mode 1 (top) and for separate sets of EOF predictors at each time-to-recurvature group (bottom). Recurvers (dotted), straight-movers (dashed) and the total correctly classified in both storm categories (solid) are indicated.

matrix scores for the dependent clean set (161 cases) and for the learning set cases not belonging to any of the clean set storms (458) are presented in Table 7.

Table 6. EOF MODE SELECTION CRITERIA FOR EUCLIDEAN METHOD TIME-STEP PREDICTOR SETS: Sets are formed from the stepwise selection of the first 45 EOF modes in the order (column 2) of their individual skill in distinguishing between clean set recurvers and straight-movers (predictability) and or simply in numerical order. A mode is selected (column 3) if the new set skill is greater than (GT), or greater than or equal to (GE) the skill before the addition of that mode. This stepwise process is continued until all 45 modes are tested. Then, the total number of predictors retained in the set is limited to the number specified in column 4. "NONE" indicates that no restriction is placed on the total number of predictors that may be retained in the set. "10 (MIN)" indicates that only the minimum number of modes required to achieve the same skill as the first ten modes selected in the stepwise selection process are ultimately retained in the time-step set.

SELECTION CRITERIA	SELECTION ORDER	MODES RETAINED IF NEW SET SKILL IS GE OR GT BEFORE	LIMIT
A	PREDICTABILITY	GT	NONE
B	PREDICTABILITY	GE	10
C	PREDICTABILITY	GE	10 (MIN)
D	NUMERICAL	GE	15
E	NUMERICAL	GE	10 (MIN)

The stability of the Euclidean model is judged first by comparing the D-score for the independent and dependent samples. This D-score evaluates only the categories R-00h through R-96h plus NR that comprise the dependent sample. As expected, skill is best (D-score = 1.78-2.07) for the dependent set classifications. The degradation in the D-scores for the independent sample, which range from 2.56 to 2.73, is not linear. For example, the second-best score for the independent sample (2.57) is for a model that has the worst D-score (2.07) for the dependent sample. In addition, the model with the best D-score (1.78) in the dependent sample has one of the worst D-scores with the independent sample. Notice that higher skill is attained when the selection of the EOF predictors is according to their relative predictability (lines 1-6) than if selection is simply in numerical order (lines 7-9). However, the selection of such a large number of EOF predictors, and especially the selection of such high order modes as 41 and 42, is a

Table 7. CLASSIFICATION SKILL FOR TWO METHODS OF SELECTING EUCLIDEAN MODEL PREDICTORS: 700 mb classification skill in terms of D- and I-scores for the independent set forecasts (first two columns) and D-scores for the dependent (clean set) forecasts. Predictor EOF modes in lines 1-9 are from various subjective combinations of the sets of predictors that separate the individual time-to-recurvature groups (R-00h through R-96h) from the PRNR group. Selection criteria for these time-step sets (column 4) are explained in Table 6, and the number in parentheses indicates the number of R-00h through R-96h sets in which an individual mode must have appeared to be retained in the potential overall best set. R-00h (lines 10 and 11) time-step sets are also evaluated as Euclidean model predictors.

INDEPENDENT: D-SCORE	INDEPENDENT: I-SCORE	DEPENDENT: D-SCORE	TIME-STEP SET SELECTION CRITERIA	MODES:
SETS OF MODES COMMON TO R-00H THROUGH R-96 SETS:				
2.63	2.52	1.92	A (3)	1 4 10 30 31 34
2.64	2.52	1.84	B (3)	1 4 8 10 20 23 25 30 31 34 37 38 41 42
2.58	2.52	1.81	B (4)	1 4 20 25 30 31 34 37 41 42
2.56	2.46	1.85	C (3)	1 4 10 20 25 30 31 34 38 41 42
2.68	2.56	1.92	D (8)	1 4 7 10 12 13 15
2.57	2.50	2.07	D (9)	1 4 12
2.72	2.54	1.78	E (3)	1 2 3 4 5 7 8 9 10 12
2.73	2.60	1.89	E (6)	1 2 3 4 5 7
2.69	2.60	2.00	E (7)	1 4 7
R-00H SETS:				
2.55	2.40	2.27	A	1
2.48	2.41	1.87	C	1 31 20 34 24 42 23 4

concern. It may be that the dependent set is being well described, but this is at the expense of degradation in the independent sample performance.

Another subjective, but physically based, approach can be used in the selection of predictors for the Euclidean method. Recall that the time-to-recurvature coefficients in Fig. 4 trace out a smooth path in the EOF 1 - EOF 2 domain. Since these coefficients are in time order, increasing the geometric distance between the beginning and end time coefficients should also increase the distances between the intermediate time values. Thus, the hypothesis is that the set of predictors that best distinguishes between the R-00h and NR clean set cases may also be best for identifying the intermediate 12-h time-to-recurvature cases. To test this hypothesis, the R-00h sets formed using the stepwise selection criterion A and C in Table 6 are evaluated as overall Euclidean model predictor sets (bottom group, Table 7). Both sets are selected from the first 45 EOF modes in order of their individual predictability. As indicated above, only EOF Mode

1 enters the 700 mb model if selection criteria A is applied. If the mode retention criterion is relaxed to just greater or equal (selection criteria C), seven additional modes are included in the set. These additional modes significantly improve dependent sample classification skill (D-score = 1.87 versus 2.27 obtained using Mode 1 alone). Rated on the D-score performance, independent sample classification skill is also higher (2.48 versus 2.55 for Mode 1 alone). Surprisingly, the I-score for the independent sample classifications is slightly less (2.41 versus 2.40) for Mode 1 alone. This indicates that the independent sample PR cases, included only in the I-score, must be well classified using EOF Mode 1 alone. Both Euclidean models based on the R-00h sets demonstrate higher skill in classifying the independent sample (D-score = 2.48-2.55 and I-score = 2.40-2.41) than those models based on the predictors common to all R-00h through R-96 sets (D-score = 2.56 - 2.73 and I-score = 2.52-2.60). Therefore, the conclusion from these tests is that the EOF modes that best distinguish between the R-00h and straight-mover cases also provide the best Euclidean model skill in identifying the correct 12-h time-to-recurvature (R-00h through R-96h) or non-recurvature (PRNR) forecast group.

Based on the above conclusions, the search for an overall best set of predictors for the Euclidean model is confined to those sets that provide the best distinction between the clean set R-00h and straight-mover cases. Since the problem is reduced to the separation of only two categories of data, univariate hypothesis testing can be used to identify the modes with the greatest difference between R-00h and NR mean values. Individually these modes provide the greatest separation between the R-00h and PRNR groups in one-dimensional space. Therefore, some combinations of these modes also should provide the best separation of the R-00h and PRNR classification groups in multidimensional EOF space. An EOF mode is identified as having significantly different R-00h and NR means if the p-value for a two sample t-test of the clean set R-00h and NR coefficients for that EOF mode is less than or equal to 0.01. Since the p-value is the smallest significance value at which the null hypothesis (that the R-00h and NR means are equal) can be rejected, this test objectively identifies the modes with the greatest separation between the R-00h and NR means (Fig. 12). As expected, the largest difference between the mean EOF values for the R-00h and NR groups at 700 mb is for Mode 1. Ten other modes also have significant differences in mean EOF values according to this test. Overall predictor sets are then chosen from among these significant modes using the stepwise selection criteria described in Table 8.

Euclidean model skill in identifying recurvers and straight-movers is compared in Fig. 13 for R-00h predictors selected from only the significant modes (top) and for the

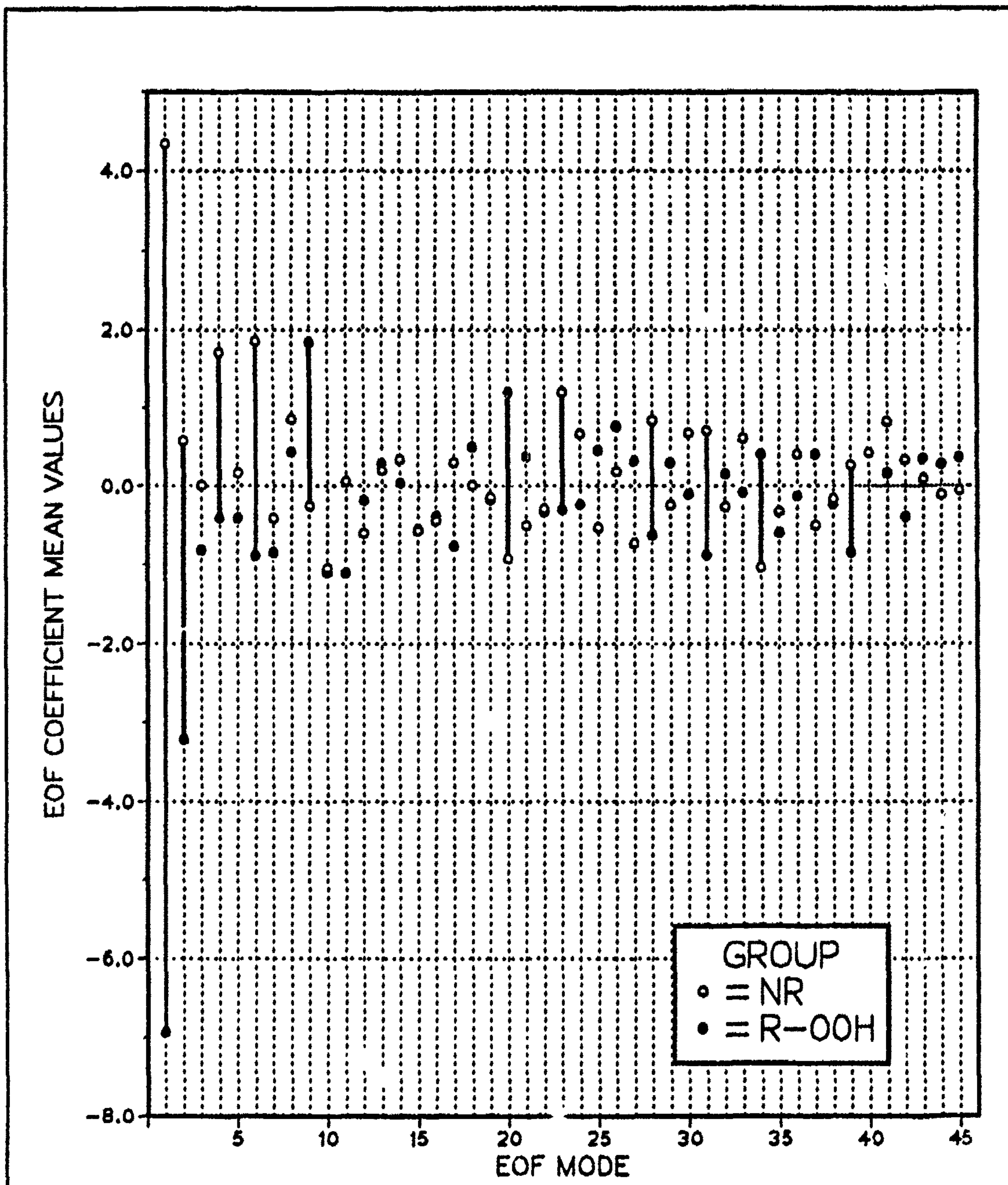

Fig. 12. Significance testing for Euclidean clean set R-00h versus NR means. Solid vertical bars indicate the 700 mb EOF modes (abscissa) that have significantly (p-value $\leq$ 0.01) different R-00h and NR mean coefficient values (ordinate) based on a two sample t-test.

same number (eight) of predictors selected from among the 45 EOF modes (bottom). Both of these sets demonstrates better skill in distinguishing recurvers and

Table 8. SIGNIFICANT MODE SELECTION CRITERIA FOR R-00H PREDICTOR SETS: Criteria as in Table 6, except applied only to those modes identified by a two sample t-test as having significantly (p-value ≤ 0.01) different R-00h and NR mean coefficient values.

SELECTION CRITERIA	SELECTION ORDER	MODES RETAINED IF NEW SET SKILL IS GE OR GT BEFORE	LIMIT
F	NUMERICAL	GE	NONE (MIN)
G	NUMERICAL	GE	NONE
H	NUMERICAL	GT	NONE
I	PREDICTABILITY	GE	NONE (MIN)
J	PREDICTABILITY	GT	NONE

straight-movers than the Mode 1 model in Fig. 11 (top), and less skill in the R-36h through R-96h periods for the optimum time-step model in Fig 11 (bottom). One advantage of the model based on the significant modes is that the separate levels of skill for recurvers and straight-movers a e more consistent. By contrast, the nearly equal combined skill for the numerical Eι F mode model is gained by much better skill for straight-movers than for recurvers.

The final step in the Euclidean distance model development is then to evaluate the R-00h predictor sets (F through J in Table 8) and identify the set and pressure level with the highest time-to-recurvature classification skill. The classification matrix scores for the independent and dependent sample classifications for the EOF modes selected on the basis of hypothesis tests are presented in Table 9.

Even though the number of EOF modes is limited by the significance testing, the selection criteria in Table 8 can lead to different Euclidean models. Except for the 700 mb Mode 1 model, the largest sets of predictors are selected at 700 mb (6-8 modes) and 400 mb (4-8 modes). The 250 mb model using only two or five modes demonstrate the best skill in identifying the 12-h time-to-recurvature groups in the dependent sample (250 mb D-score = 1.81-1.83, 400 mb D-score = 1.81-1.92 and 700 mb D-score = 1.93-2.27). As noted previously for the Euclidean models in Table 7, the degradation in the D-score for the independent samples typically is not linear. For the independent sample, the skill for 250 mb (D-score = 2.43-2.45 and I-score = 2.40-2.45) and 700 mb (D-score = 2.44-2.55 and I-score = 2.36-2.40) are nearly comparable. Less skill is

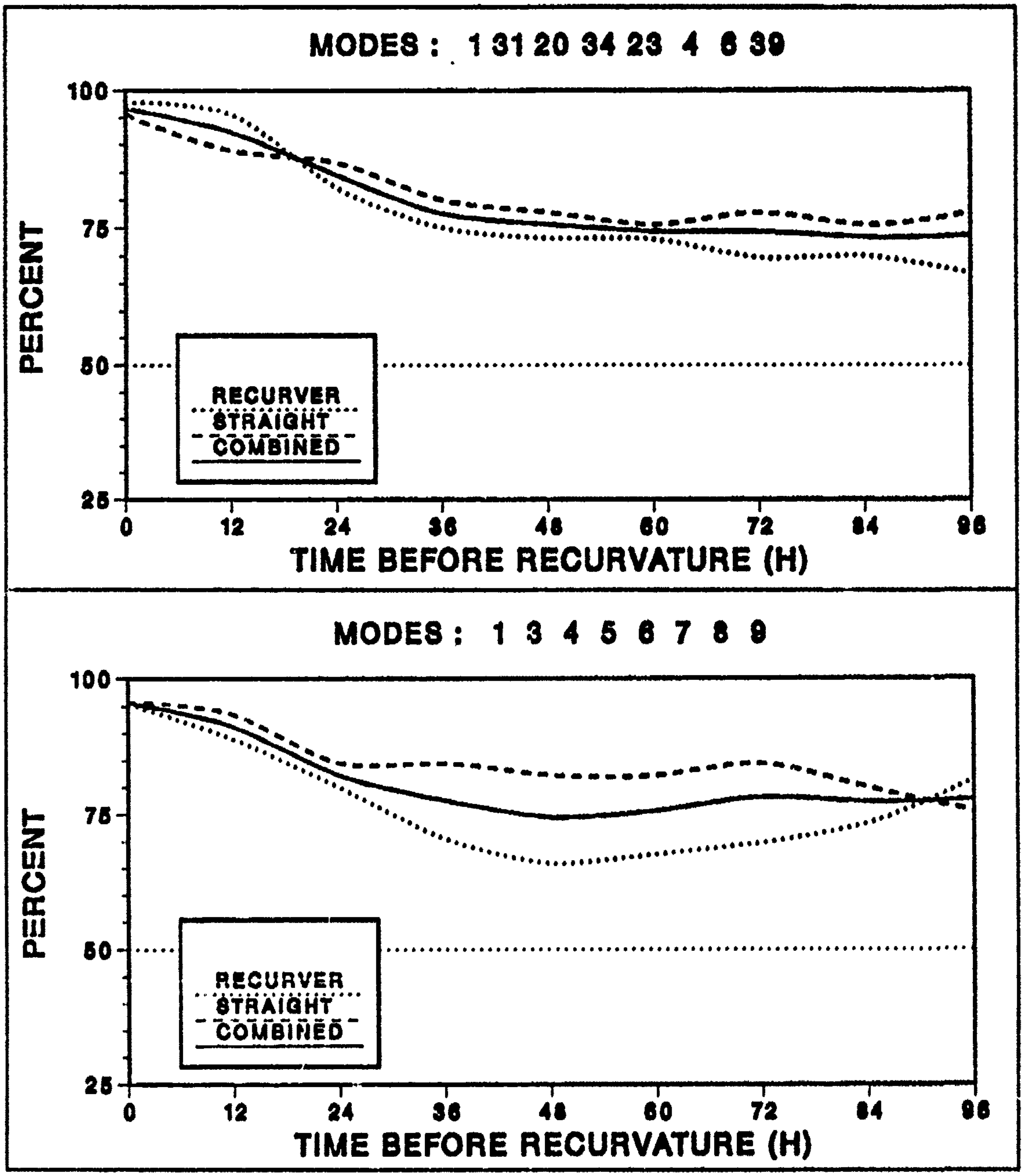

Fig. 13. Euclidean method classification skill using R-00h sets of EOF predictors. Classification skill for 700 mb as in Fig. 10, except for a R-00h set chosen (selection criteria I in Table 8) from only those modes identified with significantly different R-00h and NR clean set EOF mean coefficient values according to a two sample t-test (top), and for a R-00h set of a similar number (eight) of modes selected (similar to criteria D in Table 6, except limited to eight vice ten predictors) from all 45 EOF modes (bottom).

Table 9. CLASSIFICATION SKILL FOR EUCLIDEAN MODELS AT THREE PRESSURE LEVELS: Classification skill as in Table 7 for the five selection criteria F through J (described in Table 8) of EOF mode predictors at each pressure level with significantly different (two sample t-test p-value ≤ 0.01) clean set R-00h and NR mean EOF coefficient values.

INDEPENDENT: D-SCORE	INDEPENDENT: I-SCORE	DEPENDENT: D-SCORE	LEVEL	SELECTION CRITERIA	MODES:
2.50	2.39	2.01	700	F	1 4 6 20 23 28
2.47	2.37	1.94	700	G	1 4 6 20 23 28 34
2.55	2.40	2.27	700	H	1
2.44	2.36	1.93	700	I	1 31 20 34 23 4 6 39
2.55	2.40	2.27	700	J	1
2.67	2.62	1.88	400	F	1 2 4 6 10 13
2.62	2.55	1.83	400	G	1 2 4 6 10 13 14 18
2.56	2.50	1.91	400	H	1 2 4 10
2.61	2.49	1.81	400	I	1 4 6 14 18 2
2.66	2.55	1.92	400	J	1 4 6 2
2.45	2.40	1.81	250	F	1 6
2.43	2.45	1.83	250	G	1 6 10 12 15
2.45	2.40	1.81	250	H	1 6
2.43	2.45	1.83	250	I	1 6 15 10 12
2.45	2.40	1.81	250	J	1 6

noted at 400 mb (D-score = 2.56-2.67 and I-score = 2.49-2.62). These sets of Euclidean model predictors identified by significance testing tend to outperform the R-00h sets selected (criteria A through E in Table 6) from all 45 Modes 1-45 (not shown), unless by chance they contain the same modes.

Judged on the independent sample classification matrix D-scores, the best Euclidean distance model using the 250 mb vorticity includes EOF Modes 1, 6, 10, 12 and 15 (lines 12 and 14). Two advantages of this set are that only five predictor variables are required and no EOF mode greater than 15 is included. By contrast, the best 700 mb set selected using criteria I has eight EOF modes, and includes higher order modes such as 31, 34 and 39.

C. MODEL EVALUATION

The final Euclidean model at 250 mb is evaluated in terms of skill in classifying the learning set of 782 cases (Table 10). The combined skill in correctly identifying recurvers (75%) and straight-movers (68%) during the 72-h forecast period is 71%. This compares with %R, %S and %T scores of 36, 64 and 54 for climatology (Table 5). Skill in

identifying the time to recurvature is best near recurvature (R-00h = 45%, R-12h = 21% and R-24h = 35%), and in the straight-track categories (R-96h = 38% and PRNR = 35%). The higher skill at the ends of the forecast interval may be because the EOF predictor modes were selected to achieve maximum separation of the R-00h and the NR mean EOF coefficients. In addition, there may be more variability in the vorticity fields as recurvature conditions develop (R-36h through R-84h).

Table 10. CLASSIFICATION MATRIX FOR FINAL EUCLIDEAN MODEL: Classifications for observations in each 12-h verification category and the percent correctly forecast by the 250 mb Euclidean model. Percent of recurvers and straight-movers correctly predicted is also listed.

MODES: 1 6 10 12 15

			CLASSIFICATION									
	VERIFY	CORRECT	00	12	24	36	48	60	72	84	96	PRNR
RECURVER: (75%)	R-00H	(45%)	25	9	11	4	1	1	1	0	1	2
	R-12H	(21%)	14	12	13	7	3	0	0	1	3	3
	R-24H	(35%)	4	9	19	7	3	3	1	0	5	4
	R-36H	(17%)	3	3	9	9	8	3	2	2	5	8
	R-48H	(22%)	1	2	3	9	10	2	3	2	7	7
	R-60H	(12%)	1	0	2	7	2	5	7	1	10	6
	R-72H	(13%)	1	0	1	3	3	3	4	2	8	7
STRAIGHT: (68%)	R-84H	(10%)	0	0	1	3	1	6	0	3	6	10
	R-96H	(38%)	0	0	0	1	4	2	5	0	9	3
	PRNR	(35%)	1	5	16	24	28	23	22	17	120	135
TOTAL (71%)												

Bar charts (Fig. 14) of the percent of learning set cases in each 12-h verification category that are classified into each time-to-recurvature group further confirm the relatively poor ability of the Euclidean method for the R-84h through R-36h cases. The intermediate 12-h categories not shown in Fig. 14 tend to have similar characteristics as the 24-h bar charts. Times near recurvature and in the straight-track categories are better classified and are also more likely to be classified within only one or two classification groups of the correct value. Cases in the intermediate forecast intervals (R-36h through R-84h) are more likely to be misclassified, and the classification errors, in terms of the number of 12-h categories between the forecast and the verification groups, are greater.

The Euclidean model classifications presented above have higher skill than the climatological forecasts of the learning set cases (Table 5). For example, the I- and

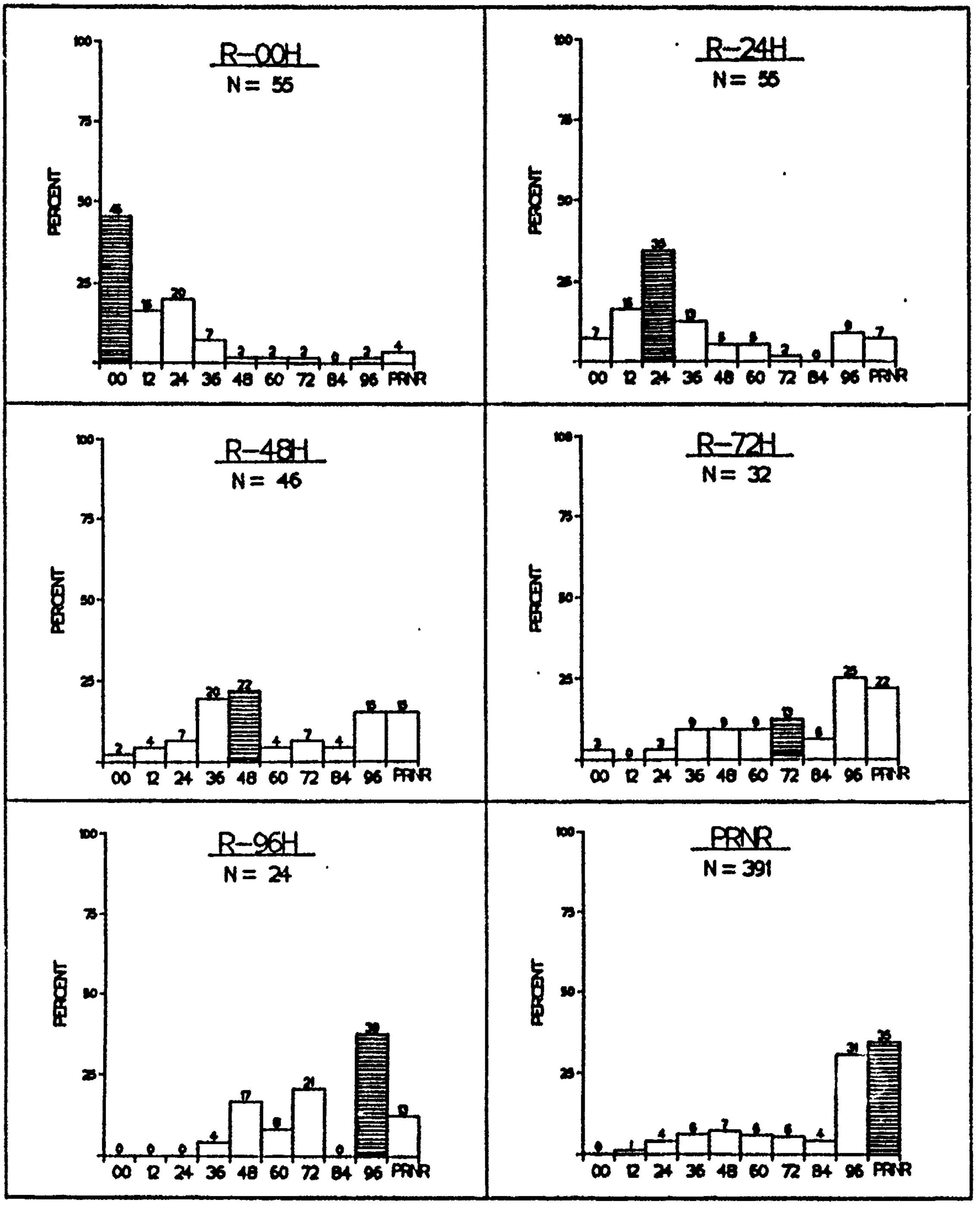

Fig. 14. Classification bar charts at 24-h intervals for the Euclidean model. Percent of N cases (ordinate) verifying as R-00h (top left), R-24h (top right), R-48h (middle left), R-72h (middle right), R-96h (bottom left), and PRNR (bottom right) that are classified into each group R-00h through PRNR (abscissa). Shaded bars indicate the percent in the correctly classified category.

R-scores of these Euclidean model 12-h forecasts are 2.34 and 2.10 versus 3.93 and 5.30 for the climatological forecasts, respectfully. However, the skill in identifying the time to recurvature is less than desired for operational use. Because of the subjectivity in the development of the Euclidean model, these results should not be used to make final conclusions regarding the usefulness of an EOF representation of vorticity to forecast tropical cyclone recurvature. No definitive method was found for selecting the optimum set of EOF predictors in the Euclidean method. In addition, each EOF mode that is selected is given the same weighting, rather than assigning additional influence to the modes that have the most significance. Furthermore, the use of a small clean set of storms in this approach may not provide the most robust definition of the time-to-recurvature classification groups. Nevertheless, the Euclidean method has easily understood physical interpretation for using an EOF approach in identifying the vorticity patterns associated with recurvature. The above results indicate the approximate levels of skill that can be expected using these predictors. However, a more objective approach is needed to identify the optimum set of EOF predictors and to better exploit the relative contributions of each mode in the recurvature forecast model.

IV. DISCRIMINANT ANALYSIS APPROACH

The approach in this section is to use discriminant analysis techniques to better exploit the predictive skill of EOF coefficients of vorticity in forecasting tropical cyclone recurvature. The UCLA Biomedical Computer Program BMDP7M (Dixon 1988) is used to select the predictors and develop the discriminant analysis model. Although discriminant analysis is a seemingly more objective approach than the Euclidean distance method, the user must still make many choices both in its application and evaluation. Searching for the optimum discriminant analysis model requires extensive testing and should be conducted on a much larger sample population. Thus, the goal of this study is to isolate a justifiable prediction model that indicates the potential of this method.

A. DISCRIMINANT ANALYSIS

Discriminant analysis is a statistical procedure for identifying the boundaries between groups in terms of the variable characteristics that distinguish one group from another. It is used to classify cases into one of several groups and to examine the relative contributions of one or more variables in distinguishing between groups. The procedure was first introduced by Fisher (1936). The Fisher discriminant function has the form

$$Z = a_1X_1 + a_2X_2 + ... + a_pX_p, \tag{4.1}$$

where Z is the discriminant score, $X_1, X_2, ...X_p$ are the values of each predictor and $a_1, a_2, ...a_p$ are coefficients that, if standardized by pooled standard deviations, give an indication of the relative weight of each predictor. Discriminant functions are derived such that the differences in discriminant scores or the relative distances between groups are maximized. The first function separates the members of the most distinguishable group, K_1, from the remainder of the groups, K_2 through K_g. The second discriminant function separates the next most recognizable group, K_2, from the remaining groups, K_3 through K_g. The number of functions required is one less than the total number of groups, g. For each discriminant function, a cutoff score is found by taking the mean of the average score for all cases in the group K_i and the average score for all cases in the remainder of the groups K_{i-1} through K_g. An individual case is classified into group K_i if its discriminant score Z_i is greater than the cutoff score for the first discriminant

function. If the discriminant score is less than the cutoff, a second discriminant score is calculated using the second discriminant function. The second discriminant score is compared to a second cutoff score to determine if the case is in group K_2 or the remaining groups K_3 through K_g. The process continues until the case is classified.

A simpler adaptation of Fisher's classification procedure is used in statistical packages such as BMPD7M (Klecka 1980 and Dixon 1988). A classification function for each group is derived as a linear combination of coefficients and predictors plus a constant term. Predictors can be specified or they can be selected in a stepwise fashion based on user-specified criteria. To determine group membership, each function is evaluated using the predictor values of the test case to obtain a classification function score for each group. The case is classified into the group for which it has the highest classification score.

Classification function coefficients cannot be standardized and interpreted in the same manner as discriminant function coefficients because there is a different function for each group. However, discriminant functions can be computed from classification functions to examine the relationship between predictors and group classification (Afifi and Clark 1984). More commonly, statistics derived from canonical correlation analysis techniques are used for this purpose. Canonical correlation analysis examines the linear relationship between independent variables (predictors) and one or more sets of dependent variables (groups). A linear combination of predictors called a canonical variable or canonical discriminant function is formed that provides the best separation among groups. Second and subsequent canonical discriminant functions are then formed that are orthogonal and best separate the groups on the basis of associations not used in the preceding canonical discriminant functions. The maximum number of canonical discriminant functions is equal to the number of groups minus one or the number of predictor variables, whichever is less. Canonical discriminant functions can also be used to classify. Final classifications will generally be identical to those obtained with classification functions unless the group covariance matrices are not equal (Klecka 1980). A complete discussion of the application of canonical correlation statistics to discriminant analysis can be found in Klecka (1980) or Afifi and Clark (1984).

B. MODEL ISSUES

Several issues basic to the development of a discriminant analysis model are considered in this section. These issues include the selection of a dependent sample, how far

in advance recurvature can be recognized, and the optimum number and composition of the classification groups. Decisions in these areas are based on the ability of EOF modes to predict recurvature as well as the classification goals of the forecast model. These decisions, in combination with choices in the application of the discriminant analysis method, will affect the level of classification skill that can be achieved with a given set of predictors and predictands. Only 250 mb data are considered in this section, because the data for this level provided the best discriminating power in the Euclidean distance approach and in comparative tests (not shown) using discriminant analysis.

1. Dependent Sample Selection

Ideally, the sample population should be divided into dependent and independent subsets to permit validation of the discriminant analysis classification model. Classification functions may be fit well to a small dependent sample, but not be effective in predicting an independent sample. Independent testing is thus necessary to better estimate the ability to correctly predict the total population. Opinions vary on the appropriate sizes of the subsets. However, the dependent subset must be sufficiently large to ensure the stability of the classification function coefficients (Klecka 1980).

Several aspects of the discriminant analysis must be specified to test the effect of the dependent subset options. As a first test, the classification groups will be the same ten categories as in the Euclidean distance approach: recurvature time to recurvature time minus 96 h in 12-h increments plus the non-recurvers. Although only straight-mover storm data are used to describe the non-recurver group while developing the discriminant analysis model, later tests will consider the observations more than 96-h prior to recurvature as part of the straight-mover set. Classification functions are derived from predictors selected in a stepwise fashion using a common F-to-enter value of 2.5. Although dependent subsets vary in size from 158 to 510 cases, this F-to-enter value is significant at better than the 99th percentile for all subsets. Therefore, differences in predictors selected in the discriminant analysis procedure can be mainly attributed to statistical differences among the dependent subsets. The classification functions then are used to classify both the dependent subset and the remaining independent cases including pre-recurver cases. Classification matrix scores (described in Section II.C.2), are computed for dependent and independent subset classifications separately and for the entire sample classifications.

The purpose of the intercomparison of the classification models derived from 13 different dependent subsets of the 250 mb sample (Table 11) is to test the stability of the classification functions. Whole-storm data from the same set of clean recurving

and straight-moving storms in the Euclidean distance approach are used to form the first dependent subset. Since EOF coefficients tend to progress in a similar manner as storms approach recurvature, analyzing a subset comprised of entire storms may lend statistical stability to the analysis. Two other whole-storm dependent subsets are formed from all 1979 to 1982 storms and from a random selection of two-thirds of the storms in the sample population. To test the stability of these classification functions, ten dependent subsets are formed by random selection of two-thirds of the cases in the sample population.

Table 11. **DEPENDENT SUBSET SELECTION:** Stepwise discriminant analysis of the times to recurvature for 13 dependent/independent subsets of 250 mb vorticity EOF coefficients, which are indicated in the order they were selected.

INDEPENDENT D-SCORE	DEPENDENT D-SCORE	COMBINED I-SCORE	DEPENDENT SUBSET	N	PREDICTORS (EOF MODES):
2.86	1.99	2.59	CLEAN STORMS	158	1 19
2.35	2.43	2.40	79-82 STORMS	510	1 2 3 5 36 6 45
2.62	1.99	2.17	RANDOM STORMS	449	1 3 2 5 41 7 6
2.47	2.37	2.41	RANDOM CASES 1	442	1 2 3 5 15
2.27	2.20	2.23	RANDOM CASES 2	457	1 2 3 5 24 6 41
2.32	2.08	2.15	RANDOM CASES 3	454	1 2 5 3 9 6 24
2.49	2.35	2.36	RANDOM CASES 4	443	1 2 3 5 6
2.50	2.08	2.25	RANDOM CASES 5	427	1 5 2 3 6 24 23
2.55	2.48	2.50	RANDOM CASES 6	435	1 3 2 5 45
2.38	2.24	2.31	RANDOM CASES 7	452	1 2 3 5 41 14
2.33	2.23	2.22	RANDOM CASES 8	451	1 3 5 14
2.11	2.15	2.10	RANDOM CASES 9	426	1 2 5 7 24 6
2.12	2.11	2.08	RANDOM CASES10	435	1 2 5 24 6 4

The results in Table 11 reflect differences due to the independent sample composition as well as to the dependent sample composition. If the classification functions were very stable, the various methods of subsampling in Table 11 should have involved the same predictors and have nearly equivalent dependent-independent verification scores. In practice, predictors vary in number, modes and in the order selected. This order is not necessarily indicative of their relative importance because a strong discriminator may be selected late or not at all in a stepwise analysis if the intercorrelation with other variables reduces its unique contribution to the analysis. Mode 1 EOF coefficient is the only predictor selected for all 13 dependent subsets tested. Modes 5, 2 and 3 are selected in 12, 11 and 10 of the subsets respectively. Since Mode 6 appears in

eight of the subsets, it is potentially important in the discriminant analysis. Notice that Mode 4 is selected only once, and that many higher order modes are selected after Mode 6.

Classification matrix scores also vary. Classification functions derived for the clean storm sample in Table 11 demonstrate a high degree of skill in classifying the dependent subset cases, but perform poorly in classifying independent subset cases. The combined classification matrix score for this subset pair is much worse than for other pairs, which indicates that the model is well-fitted to the dependent subset only and is not accurate on an independent subset. This result may suggest a flaw in the use of the clean storm set for the Euclidean method, where the excellent distinction in the dependent set was not sustained in the remaining cases.

The independent test results can be overly optimistic if the subset contains a disproportionate number of cases that are statistically easy to classify. For example, the classification functions derived from 1979-1982 storm data demonstrate better skill in classifying the independent subset (1983-1984) than the dependent subset. This unexpected result can be explained by examining the differences in the storms between the two subsets. Patrick Harr (personal communication) found that western North Pacific tropical cyclones during 1983 and 1984 had recurvature tracks that were similar to climatology and were relatively easy to forecast in comparison to those in the previous four years.

The 10 random subsets in Table 11 were generated to test whether the classification functions derived from dependent subset predictors would work equally well on the independent subset. In other words, the randomly selected cases should have nearly equal classification matrix scores. Only the two subset pairs formed from the ninth and tenth randomly selected cases have nearly equal dependent and independent classification matrix scores. Because the classification model derived from randomly selected storms (line 3 in Table 11) outperforms those derived from randomly selected cases in dependent subset classification, retaining data from entire storms in the dependent set may aid in the derivation of skillful classification functions. However, the marked degradation in the independent D-score for the random storm independent subset indicates that the differences in skill are also a function of which subset contains more storms that are inherently easier to classify.

The conclusion from Table 11 is that classification functions derived from randomly selected subsamples of this data set are not statistically stable. To improve the stability, the entire sample population will later be used to both derive and test

classification functions. In lieu of independent testing, jackknifing is employed to assess the degradation in classification skill expected in the total population. In this procedure, N sets of classification functions are derived by successively withholding one case from the sample of N cases. Each of the N sets of classification functions is tested on the one case that was withheld, and the summation of these verifications is an indication of the likely accuracy of a single discriminant analysis based on the entire sample. Although jackknifed results are computed for each discriminant analysis, they will be presented only in the selection and testing of an optimal classification model for this sample of storms (Sections IV.D and IV.E.1).

2. Limits of discrimination for time to recurvature

A basic question is the limitation of the discriminant analysis to separate the EOF coefficients associated with storms more than 96 h before recurvature from the straight-mover coefficients. To illustrate this limitation, univariate statistics for EOF Mode 1 coefficients are compared. Mode 1 not only accounts for the largest percent of the variance in the synoptic vorticity patterns, but also demonstrates the greatest predictive capability. It is the only predictor consistently selected and is selected first in all subset analyses (Table 11).

Distributions of the Mode 1 means, 95% confidence intervals and the standard deviations for times R-00h through R-96h in 12-h increments, plus the pre-recurvers (PR) and non-recurvers (NR) in the entire 250 mb data set are presented in Fig. 15. Univariate statistics for a combined PR and NR group are also plotted. Group statistics can be interpreted in terms of the physical processes they represent. Recall that the pattern for Mode 1 (Fig. 2) is representative of the vorticity pattern associated with a storm in the monsoon trough and that the magnitude of the coefficient is indicative of the importance of the pattern (or the opposite pattern if it is negative). Group means vary almost linearly from large positive values for non-recurver and pre-recurver situations to large negative values at recurvature. Variances are large and the considerable overlap among groups indicates the variability in vorticity patterns that lead to recurvature. These group means are most separated in the 36 h preceding recurvature. However, variances are also largest during these times. These large differences are associated with rapid changes in the storm-centered vorticity patterns accompanying storms moving around the subtropical ridge. In contrast, vorticity patterns change little for storms moving along the monsoon trough well prior to recurvature.

The challenge for the discriminant analysis (or the Euclidean method) is to distinguish those EOF modes that best indicate the time to recurvature. With the similar

group means for NR, PR, R-96h and R-84h times, it is unlikely that the discriminant analysis could consistently separate these groups from Mode 1 only. The NR group mean is slightly smaller than the group means for pre-recurvers and the R-96h cases. Combining non-recurver and pre-recurver subsamples provides a smoother and more physically plausible transition among groups. That is, the R-96h and R-84h samples might also have been added to the new PR and NR group. However, since the official JTWC forecast period is 72 h, retaining the R-96h and R-84h as separate classification categories provides a forecast 'buffer'. The R-96h and R-84h predictions provide an alert of a trend toward recurvature, but not within the current 72-h forecast period. Statistically, these intermediate groups decrease the likelihood that non-recurvature situations will be misclassified into the next similar group, and thus prompt the forecaster to erroneously predict recurvature within the 72-h forecast period.

Based on the above considerations, the PR sample is combined with the straight-mover sample to define the PRNR classification group. The merits of other data combinations are better assessed in terms of gains in classifiability versus loss of time resolution. These issues are explored in the next section.

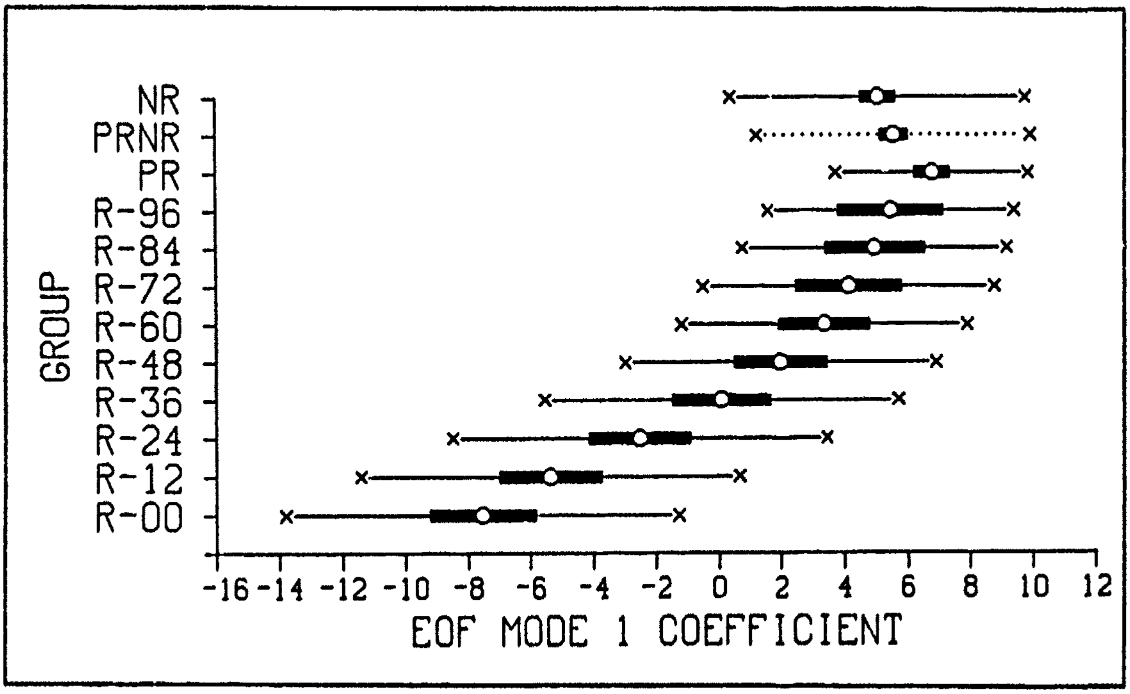

Fig. 15. Univariate statistics for 250 mb vorticity EOF Mode 1. Mean (open circle), 95% confidence interval (solid bar) and standard deviation (x) for individual groups (solid line) and for the combined PR and NR group data (dotted line).

3. Number and composition of classification groups

The analysis goal is to fully utilize the time resolution of the data set to predict the time to recurvature in 12-h increments. However, the EOF coefficients for the vorticity fields may not have enough discriminating power to reliably discern between synoptic situations with this time resolution. Perhaps these predictors would be suited to classify some combination of groups with decreased time resolution. Thus, combinations of groups are tested to increase the percent of correct classifications and still retain some of the time resolution desired by the forecaster.

The univariate distributions of EOF Mode 1 coefficient for each 12-h group in Fig. 15 indicate that this predictor alone cannot adequately discriminate among neighboring groups. Other EOF modes may provide additional dimensions that distinguish differences among the 12-h groups. The effect of multiple predictors and the effect of combining groups on classification skill is best examined by discriminant analyses and canonical correlation statistics.

To evaluate the trade-off between time resolution and forecast accuracy, combinations of time groups are tested that are potentially easier to classify and are still useful to the forecaster. Stepwise discriminant analysis is performed using F-to-enter values significant at the 0.01 level for the sample size. Analysis models with two, three and ten classification groups are compared in Tables 12, 13 and 14. Verifications are identified by their 12-h data categories so that the loss of time resolution in the classification groups can be appreciated. Pre-recurver data are combined with non-recurver data for both classifications and verifications.

The minimum useful distinction for the forecaster is between recurving and straight-moving storms. The recurver group is defined by R-00h through R-72h samples, and the straight-moving group is defined by R-84h through pre-recurver and straight-mover data. Thus, a successful prediction would identify either a recurving or straight track during the current 72-h forecast period. The two-group discriminant analysis (Table 12) correctly identifies R-00h to R-72h cases as recurvers with 76 % accuracy. The verifications within each 12-h category do not have the same skill. The percent correctly classified decreases from 95% for cases at recurvature time to 44% at R-72H. This is because times closer to recurvature are more distinct from the straight classification group and are, therefore, more readily recognized as recurvers. Non-recurvature is correctly predicted for 81% of the R-84h through PRNR cases. The combined sample skill is 79%, but this total is dominated by skill in prediction of

straight track motion because of the larger number of non-recurver cases (445) than recurver cases (337) in the sample population.

Table 12. **TWO-GROUP DISCRIMINANT ANALYSIS MODEL:** Percent of recurvers and straight-movers in the sample population correctly predicted by the two-group discriminant analysis model with the EOF modes indicated. Number of classifications as recurvers or straight-movers are provided with 12-h time resolution to indicate at what times this analysis model succeeds or fails.

MODES: 1 3 5 36 41 24 14 38 43 6 39 27

	VERIFY	CLASSIFICATION RECURVER	CLASSIFICATION STRAIGHT
RECURVER: (76%)	R-00H	52	3
	R-12H	53	3
	R-24H	47	8
	R-36H	37	15
	R-48H	31	15
	R-60H	21	20
	R-72H	14	18
STRAIGHT: (81%)	R-84H	9	21
	R-96H	5	19
	PRNR	72	319
TOTAL (79%)			

A more useful distinction to the forecaster would be separation into high, medium and low likelihood of recurvature (Table 13). The high group is defined as the R-00h to R-24h cases, the medium group as all R-36H to R-72H cases, and the low group as the R-84h through PRNR cases. Classification functions correctly classify the sample into high, medium or low categories with 75%, 56% and 73% accuracy, respectively. While this three-group classification scheme increases the time resolution in the recurvature prediction, the ability to correctly classify track types during the 72-h forecast period (77%) is less than that in the two-group model (79%). In addition, skill in identifying straight-moving situations is degraded. The addition of another recurver group can be viewed as increasing the number of correct classification categories for recurver cases and increasing the number of incorrect classification categories for non-recurvers. The intermediate group also increases the discriminant analysis model

separation between the 12-h data categories in the high recurver group and the non-recurver categories.

Table 13. **THREE-GROUP DISCRIMINANT ANALYSIS MODEL:** Percent of recurvers and straight-movers correctly predicted by a three-group discriminant analysis model. Format is similar to Table 12.

MODES: 1 3 2 4 5 14 41 9 24 38 12 36 15 37

			CLASSIFICATION		
		VERIFY	HIGH	MED	LOW
RECURVER: (82%)	HIGH (75%)	R-00H	47	7	1
		R-12H	45	7	4
		R-24H	32	18	5
	MED (56%)	R-36H	15	28	9
		R-48H	4	28	14
		R-60H	2	24	15
		R-72H	3	16	13
STRAIGHT: (73%)	LOW (73%)	R-84H	2	11	17
		R-96H	0	6	18
		PRNR	13	89	289
TOTAL (77%)					

Discriminant analysis into the ten 12-h classification groups R-00h to R-96h plus PRNR (Table 14) maximizes the time resolution of the predictions for this data set, but at the expense of classification accuracy. The ability to distinguish between recurver and straight-track situations is only 72% as compared to the 79% and 77% accuracy achieved by the two-group (Table 12) and the three-group (Table 13) forecast models, respectively. Also, the ability to discern high, medium and low likelihood of recurvature is 2-6% less than for the three-group model. The improvement in recurver classification skill relative to skill in forecasting straight track situations between the two-group model and three-group model is again noted. The ten-group model correctly classifies recurver and straight situations with 79% and 67% accuracy, respectively. The greater difference in recurver versus straight classification skill for the two-group model (6%) than for the three-group model (2%) may be due to the increase in the ratio of the number of recurver to straight groups from 2.0 (two-group model) to 2.3 (three-group model). As expected, classification accuracy within each 12-h verification category is considerably less than for the broader high-medium-low or recurver-straight categories. Higher skill

exists in correctly classifying cases at the extremes of the forecast continuum, i.e., at recurvature (60%) and PRNR (48%). Skill in identifying cases in the intermediate categories ranges from 15% to 38%.

Table 14. **TEN-GROUP DISCRIMINANT ANALYSIS MODEL:** Percent of recurvers and straight-movers correctly predicted by a ten-group discriminant analysis model. Format is analogous to Tables 12 and 13.

MODES: 1 2 3 5 4 6 24 45 9 14 41

		VERIFY	CORRECT	00	12	24	36	48	60	72	84	96	PRNR
				CLASSIFICATION									
RECURVER:	HIGH	R-00H	(60%)	33	8	4	3	1	1	0	4	1	0
(79%)	(73%)	R-12H	(34%)	19	19	7	1	3	0	1	4	0	2
		R-24H	(26%)	5	12	14	7	4	1	4	4	2	2
	MED	R-36H	(19%)	1	6	8	10	7	2	3	6	3	6
	(53%)	R-48H	(15%)	2	2	5	9	7	4	3	2	8	4
		R-60H	(20%)	1	1	1	6	5	8	7	1	6	5
		R-72H	(38%)	1	1	0	2	2	4	12	4	2	4
STRAIGHT:	LOW	R-84H	(17%)	0	0	1	2	1	3	7	5	4	7
(67%)	(67%)	R-96H	(25%)	0	0	0	2	1	4	1	5	6	5
		PRNR	(48%)	2	3	15	24	25	25	32	31	47	187
TOTAL (72%)													

The distributions of group centroids in discriminant space (Figs. 16, 17 and 18) provide a graphic representation of the ability of the two-, three-, and ten-group discriminant analysis models to separate, and thus classify, the cases belonging to each group. Individual cases and group centroids may be located in discriminant space using canonical correlation analysis techniques discussed in Section IV.A. An x- and y-coordinate for each case is found by evaluating the first and second canonical discriminant functions, respectively, using the predictor values for the case. Group centroids are then located at the mean of the x-coordinates and the mean of the y-coordinates for all cases in the classification group. The mean coordinates of the cases in each 12-h verification category, hereafter referred to as time centroids, are also computed for the two- and three-group models (Figs. 16 and 17). Because these time centroids are equivalent to the group centroids for the ten-group model, they provide a means of comparing the relative separation achieved by each of the three discriminant analysis models.

As the 12-h time centroids are in sequential order, they reflect the time trends in the patterns accompanying recurvature. Notice that the relative separations between consecutive time centroids are generally similar along the first canonical discriminant function for all three models (Figs. 16, 17 and 18). These relative distances between group centroids indicate how well the model is able to distinguish between groups. The proximity of the 12-h time centroids to their parent group centroid gives an indication of the classification accuracy for each 12-h verification category or combination of 12-h categories.

Since the number of canonical discriminant functions computed is one less than the number of groups, a one-dimensional plot is presented for the two-group model (Fig. 16). The time centroids of the R-84h through PRNR verification categories that comprise the straight-track group are all closer to the straight-group centroid than to the recurver-group centroid. In addition, the R-72h time centroid (which belongs in the recurver group) is closer to the straight-group centroid. While the actual distribution of individual cases determines the model skill reported in Table 12, the relative positions of the time and group centroids in Fig. 16 illustrate why the model is better at correctly classifying straight track cases (81%) than it is at correctly identifying recurver cases (76%). The fitted distributions of the straight-mover and recurver cases confirm these observations. The cases in the straight group are more closely distributed about their group centroid than are the cases in the recurver group. The amount of overlap between the two distributions indicates the number of cases that may be misclassified by the two-group model. As previously noted, canonical discriminant functions can be used to classify by computing a canonical discriminant function score (not shown) to divide the cases into straight-movers and recurvers. Such a dividing line in Fig. 16 would give an exact representation of the number of cases that would be misclassified into each group by the first canonical discriminant function.

In the multiple-group discriminant analysis models, classification skill is a function of how well an individual group is separated from its neighboring groups and the actual distribution of its individual members. The three-group and ten-group model centroids (Figs. 17 and 18) are spatially separated in a curvilinear fashion that reflects a consistent time trend in the group centroids. These separation patterns explain why the classification skill is highest for the groups on either end of the time spectrum. Consider a normal distribution of sample cases for each group in an ellipsoidal pattern centered around each group centroid. For a middle group with neighbors on either side, classification skill is a function of its separation from neighboring groups and the distribution

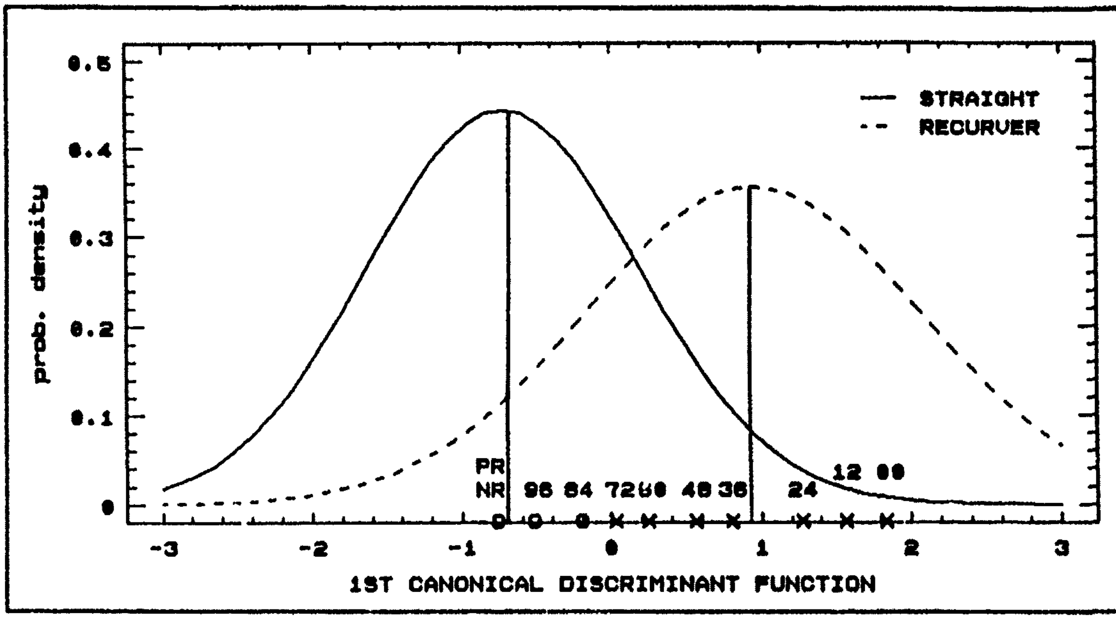

Fig. 16. Canonical discriminant function centroids for the two-group model. Group centroids are located at the mean of all cases in each group (vertical lines). Time centroids for PRNR through R-00h in 12-h intervals are indicated by (O) for categories belonging to the straight group and (X) for categories belonging to the recurver group. Fitted distributions of the straight-movers and recurvers along the first canonical discriminant function axis illustrate the overlap of the distributions.

of the individual cases belonging to the group. Since the end groups have no neighboring group on one side, sample cases on the no-neighbor side of the distribution will be classified into the end group. For both the three- and ten-group models (Tables 13 and 14), classification skill is notably higher in the end groups. Note that for the three-group model, the skill is higher for the high likelihood of recurvature group, than the low likelihood of recurvature group because the high group is better separated from the intermediate medium group than the low group. Differences in group classification skill for the ten-group model are more difficult to interpret. For example, the R-00h to R-36h group centroids are better separated in Fig. 18 than those for R-48h to R-96h. In general, better separated groups in Fig. 18 demonstrate better classification skill. One exception is the R-72h group, which has higher skill than the more separated groups and may reflect the effect of the distribution of the sample cases on classification skill.

Ultimately, the number and composition of classification groups must be a trade-off between the forecaster's need to specify a precise time of recurvature versus the diminishing skill as more precision is attempted. To illustrate this trade-off in forecast

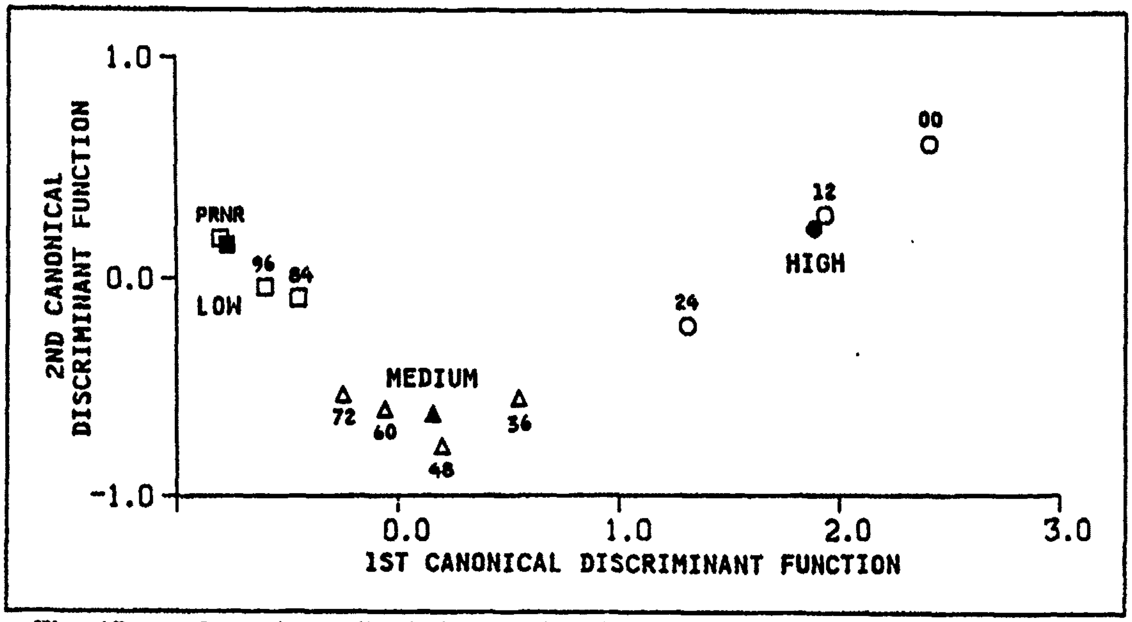

Fig. 17. Canonical discriminant function centroids for the three-group model. The first two canonical discriminant functions form the axes along which group centroids (solid markers) and 12-h time centroids (open markers) are plotted for high (circles), medium (triangles) and low (squares) likelihood of recurvature classification groups.

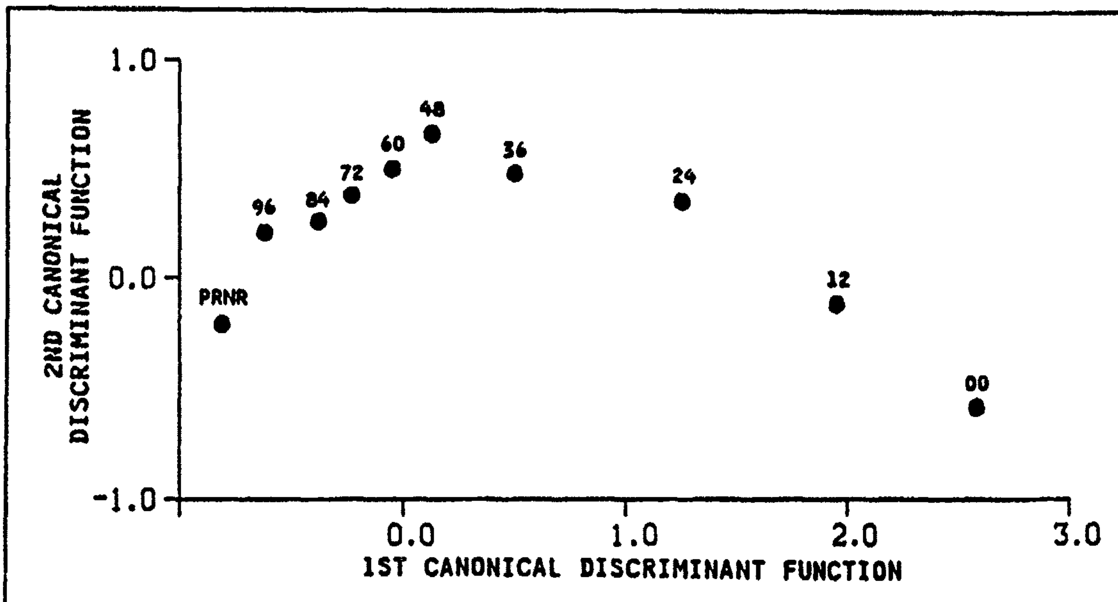

Fig. 18. Canonical discriminant function centroids for the ten-group model. The first two canonical discriminant functions form the axes along which group centroids (solid circles) are plotted for R-00h through PRNR classification groups.

accuracy and time resolution, a ten-group model will be pursued using the entire 250 mb sample population. The ten-group model is chosen to fully test the predictive capability of EOF representation of the synoptic vorticity fields to predict recurvature at the resolution of the data set.

C. APPLICATION

Discriminant analysis packages include options that permit flexibility in the selection of predictors and in the method of computing the classification functions. The optimal application of these program features is, of course, a function of the goals of the analysis. In this section, several discriminant analysis options available in BMDP7M are considered. These features include prior probabilities, contrasts and three different methods for entering predictors into the analysis. An in depth discussion of all the features available in computer packages and a comparison of BMDP7M with other discriminant analysis packages can be found in Tabachnick and Fidell (1989).

1. Adjusting for prior probabilities and the cost of misclassification

The prior probability is the probability that an individual case selected for a group is actually a member of that group. Unless otherwise specified, cases are assumed to have an equal probability of belonging to any group, and the classification functions are derived with equal probabilities of misclassification. Specifying group probabilities in the analysis procedure changes the ratio of the probability of errors by adjusting the discriminant function scores, or equivalently by adjusting the constant terms in the classification functions to achieve a ratio of errors consistent with the designated prior probabilities. Prior probabilities have the greatest effect on classification skill when groups are relatively indistinct from one another.

In this study, prior probabilities could be assigned on the basis of the group sizes in the sample or according to the climatological probability that a synoptic situation belongs to each group. For example, prior probabilities based on the relative size of each of the ten 12-h groups would range from 50% for the PRNR group to 3-7% for the remaining groups. Thus, a discriminant analysis model including these prior probabilities would only classify a case into one of the groups with a low prior probability if there was very strong evidence that it was not in the PRNR group. Prior probabilities could also be used to achieve some other desired ratio of errors that would be better suited to the needs of the forecaster. That is, if the cost of misclassification of a certain group was high, assigning a high prior probability to that group would decrease the

likelihood that a case belonging to that group would be misclassified into another group with a lower prior probability.

Assigning prior probabilities may be advantageous in future applications of discriminant analysis to fine tune the recurvature forecast model using EOF predictors. Since such adjustments to the analysis may not give a true indication of the discriminatory power of the EOF coefficients, prior probabilities will not be specified in this study.

2. Contrasts to direct stepwise selection of predictors

In discriminant analysis, a contrast is a series of coefficients, one for each classification group, that modifies the stepwise selection of predictors. The coefficient for each group indicates the relative amount of differentiation desired between groups. Coefficients must be specified such that the sum of the coefficients for all groups equals zero. Contrasts do not affect the computation of the classification functions as do posterior probabilities. Rather, contrasts affect the computation of the F-to-enter and F-to-remove statistics. Therefore, they alter the stepwise selection of predictors such that only those predictors that maximize the differences between groups are selected in the analysis. Thus, contrasts can also be used to determine which predictors are important in distinguishing between specific pairs of groups. In developing a ten-group discriminant analysis model, this is not practical because of the large number of pairwise tests to be considered. Furthermore, it may not be a sound method of selecting a final set of predictors because the predictors that are useful in distinguishing between some pairs may adversely affect discrimination between other pairs.

Several contrasts are tested in Table 15. Because F-to-enter computations are different for each analysis, stepwise selection of EOF modes is stopped after selection of the ten predictors. No contrasts are specified in the first analysis in Table 15 to allow comparison with the different contrasts. The second analysis is designed to maximize the difference between the recurvature cases and the straight-track cases. The third maximizes distinction among all recurver situations (less than 72 h) and among all straight situations equally. The fourth and fifth analyses are designed to maximize the recurvature and non-recurvature situations and also to enhance the distinction among recurvature groups.

Maximizing the difference between R-00h and PRNR groups (line 2 in Table 15) improves the overall classification skill (I-score), but skill in distinguishing time to recurvature (R-score) is less than using no contrasts. The contrast designed to increase the differences equally among all recurver and straight groups (line 3) improves

Table 15. **COMPARISON OF MODELS USING CONTRASTS:** Effect of various contrasts on predictor selection and discriminant analysis model performance. Predictor modes are in the order selected.

I-SCORE	R-SCORE	%R	%S	%T	CONTRASTS (00 THRU PRNR)	PREDICTOR MODES:									
2.1138	1.8665	77	69	72	NONE	1	2	3	5	4	6	24	45	9	14
2.0959	1.9228	78	69	73	-1,0,0,0,0,0,0,0,0,1	1	2	3	9	6	7	45	14	5	24
2.1867	1.7033	83	67	74	-3,-3,-3,-3,-3,-3,-3,7,7,7	1	3	36	2	38	14	8	21	7	34
2.0320	1.7715	82	67	73	-8,-7,-6,-5,-4,-3,-2,-1,0,36	1	3	5	41	24	14	2	9	6	38
2.2097	1.6647	85	64	73	-7,-6,-5,-4,-3,-2,-1,9,9,9	1	3	2	38	36	14	8	41	34	7

discrimination among times to recurvature (R-score), but the overall classification skill (I-score) is less than without contrasts (line 1). Contrasts designed with the additional goal of improving the ability of the model to correctly forecast the time to recurvature (lines 4 and 5) result in improvement in the time accuracy of recurver forecasts (R-score) and in the ability to recognize recurver situations (%R = 82 and 85, respectively). However, this is at the expense of weaker discrimination of straight-movers (%S = 67 and 64, respectively).

Except for EOF Mode 1, the modes selected and the order of selection vary for the various contrasts tested in Table 15. Higher mode predictors are selected earlier in those analysis models with contrasts that are designed to increase differences among multiple groups (lines 3-5) than in the analysis model using a simpler contrast between only two classification groups (line 2).

In summary, specifications of different contrasts as in Table 15 do lead to improved recurvature-related scores or straight-mover scores relative to a discriminant analysis model without contrasts. However, both scores are not improved simultaneously. Due to the complexities of a ten-group model, the changes in predictors and forecast skill produced by these contrasts are difficult to interpret. As in the specification of prior probabilities, this analysis feature may be better utilized to fine tune an EOF forecast model than to demonstrate the usefulness of EOF's in identifying recurvature situations. Since the forecast skill without the use of contrasts (line 1) is comparable to with contrasts (lines 2 through 5), the contrasts feature will not be used further in this study.

3. Direct, hierarchical and stepwise selection of predictors

Discriminant analysis model performance depends primarily on the discriminatory power of the predictor variables. When many potential predictors are available, such as the first 45 EOF coefficients for synoptic vorticity considered in this study, the question is which combination of predictors will produce the best distinction among classification groups and in what order they should enter the analysis.

Three options of selecting and entering predictors in the discriminant analysis are the direct, hierarchical and stepwise methods. In the direct method, the predictors are selected by the user and all are entered into the analysis in one step. In the hierarchical method, the user specifies both the predictors and the order they enter the analysis. The stepwise method relies on statistical criteria specified by the user to select the predictors and determine their order of entry.

The direct and hierarchical discriminant analysis methods are advantageous because they allow the user to control the predictors in the analysis. However, they require prior knowledge of the relative discriminatory value of each potential predictor. Except for EOF Mode 1, which is shown in Section II.A.2 to represent a straight-mover situation with the storm in the monsoon trough or a recurvature situation depending upon the value of the coefficient, little can be inferred about the potential discriminatory power of the increasingly complex patterns for the EOF coefficients. Therefore, a stepwise discriminant analysis will be used to select the most significant predictors.

D. FINAL MODEL DEVELOPMENT

The final model to predict time to recurvature with 12-h resolution is developed with a stepwise analysis of the entire sample population. Potential predictors are selected from the first 45 EOF coefficients representing the 250 mb synoptic vorticity fields. No prior probabilities and no contrasts are specified.

The final question is the criteria to limit the number of predictors selected in the stepwise analysis. Mathematically, the maximum number of predictors is equal to the total number of cases in the sample minus two (Klecka 1980). Although all 45 EOF coefficients could be used to develop a discriminant analysis model for these data, the objective is to obtain the best classification skill with the fewest possible predictors. Such a parsimonious solution is sought by performing ten stepwise discriminant analyses (Table 16). The first analysis is restricted to one mode, the second is restricted to two modes, and so on until ten modes are selected in the tenth analysis model. Low F-to-enter (1.0) and F-to-remove (0.996) values are specified in the analysis procedure to

ensure the selection of up to ten predictors. However, the F-to-remove values (not shown) for the modes selected in the analyses in Table 16 indicate that each of the selected modes is significant to the separation of recurver groups at the 0.01 level or better.

Table 16. **STEPWISE SELECTION OF ONE TO TEN MODES:** Classification skill in terms of I-score, R-score and percent correctly classified recurver (R-00h to R-72h) or straight (R-84h to PRNR) for ten discriminant analyses that are limited to one to ten modes successively. Jackknifed results (discussed in Section IV.B.1) reflect the skill expected with independent testing.

MODEL RESULTS:					JACKKNIFED RESULTS:						
I-SCORE	R-SCORE	%R	%S	%T	I-SCORE	R-SCORE	%R	%S	%T	STEPS	MODES:
2.32	2.24	77	66	71	2.32	2.24	77	66	71	1	1
2.29	2.23	74	69	71	2.30	2.25	74	69	71	2	1 2
2.19	2.01	78	70	73	2.22	2.06	77	70	73	3	1 2 3
2.22	2.02	75	69	72	2.27	2.07	75	69	71	4	1 2 3 5
2.24	1.95	77	67	71	2.30	2.04	76	67	71	5	1 2 3 5 4
2.24	1.91	77	64	70	2.30	1.98	76	64	69	6	1 2 3 5 4 6
2.18	1.88	80	66	72	2.26	2.00	79	65	71	7	1 2 3 5 4 6 24
2.26	1.95	77	66	71	2.36	2.09	76	64	69	8	1 2 3 5 4 6 24 45
2.17	1.89	78	67	72	2.25	2.00	77	66	71	9	1 2 3 5 4 6 24 45 9
2.11	1.87	77	69	72	2.23	2.08	75	68	71	10	1 2 3 5 4 6 24 45 9 14

An optimal forecast model is selected from Table 16 by examining gains in classification skill as the number of modes in the analysis is increased from one to ten. As expected, the jackknifed results in columns six through ten are worse than the learning model results in columns one through five. The general trend is toward improved skill (smaller penalty scores and higher percent correct classifications) as the number of modes increases from one to ten. The seven-mode discriminant analysis model best meets the analysis objectives because the addition of Mode 24 as the seventh mode improves all measures of classification skill relative to the skill for the models with six or fewer modes. However, the addition of Mode 45 as the eighth mode results in degradation in all measures of skill. The addition of Mode 9 in the nine-mode model produces only a slight improvement over the eight-mode analysis and only results in skill scores nearly equal to those for the seven-mode analysis. Some improvement is again noted

by the addition of a tenth predictor, but the %R (77) is less than the %R (80) for the seven-mode model. The seven-mode discriminant analysis model is also preferable because predominantly low-numbered EOF modes are used in the analysis. Since these lower modes represent large-scale patterns and account for a larger fraction of the variance in the synoptic vorticity fields than the higher modes, the coefficients for the lower mode EOF's should be better discriminators of recurvature than those for higher modes. Furthermore, the higher mode predictors may represent noise in the synoptic field, yet be statistically useful in predicting recurvature in this data set. Based on these considerations, the seven-mode model in Table 16 is chosen to demonstrate the potential of discriminant analysis of the EOF representation of synoptic vorticity fields to forecast time to recurvature with 12-h resolution.

E. FINAL MODEL EVALUATION

The discriminant analysis model derived in Section IV.D from the stepwise selection of seven EOF coefficients of 250 mb vorticity is indicative of the forecast skill obtainable with this analysis method and these data. In this section, the final discriminant analysis model is evaluated and compared to the Euclidean distance model derived in Section III.B. Since both the discriminant analysis and Euclidean distance models were derived from the 250 mb vorticity data, the comparison is between the two analysis methods.

1. Forecast skill

The classification matrix for the final discriminant analysis model is presented in Table 17. The model correctly identifies synoptic situations that will lead to recurvature within the next 72-h forecast period with 80% accuracy. Skill for straight-track situations is 66%. Thus, there is a greater chance of a false alarm of recurvature than a missed recurvature prediction. The combined skill in predicting track type is 72%. Group classification skill is best near recurvature (R-00h = 60%, R-12h = 29%, and R-24h = 29%) and in the straight-track categories (R-96h = 29% and PRNR = 47%). Skill in the intermediate categories only ranges from 7-22%. This result was anticipated based on initial testing with the ten-group discriminant analysis model in Section IV.B.3 (Table 14). The ten-group model in Section IV.B.3 is the eleven-mode model that would have been included in Table 16 if the stepwise selection of one to ten modes had been carried one step further. It was derived using the same analysis options from the stepwise selection of the same seven modes in the final discriminant analysis model plus Modes 45, 9, 14 and 41. Canonical discriminant functions for the final discriminant analysis model (not shown) are nearly identical to those in Fig. 18 for the ten-group

model and show relatively little separation among the centroids for the intermediate R-86h through R-36h groups.

Table 17. **CLASSIFICATION MATRIX FOR THE TEN-GROUP (SEVEN-MODE) MODEL:** Classifications for data in each 12-h verification category and the percent correctly forecast by the ten-group (seven-mode) discriminant analysis model. Percent of recurvers and straight-movers correctly predicted is also listed.

MODES: 1 2 3 5 4 6 24

			CLASSIFICATION									
	VERIFY	CORRECT	00	12	24	36	48	60	72	84	96	PRNR
RECURVER: (80%)	R-00H	(60%)	33	12	2	2	1	0	1	3	0	1
	R-12H	(29%)	17	16	10	4	3	0	1	1	2	2
	R-24H	(29%)	5	13	16	4	6	3	0	3	2	3
	R-36H	(12%)	2	8	9	6	7	3	2	6	4	5
	R-48H	(13%)	1	3	6	7	6	7	3	1	7	5
	R-60H	(22%)	2	0	1	9	3	9	6	2	3	6
	R-72H	(19%)	0	2	1	2	3	5	6	4	3	6
STRAIGHT: (66%)	R-84H	(07%)	0	0	2	1	2	6	2	2	6	9
	R-96H	(29%)	0	0	0	3	1	5	0	3	7	5
	PRNR	(47%)	2	5	16	22	31	32	21	25	53	184

TOTAL (72%)

Bar charts (Fig. 19) of the percent of cases in each 12-h verification category that are classified into each group further illustrate the relatively poor ability of the model to pinpoint the time to recurvature among the R-84h through R-36h cases. The intermediate 12-h categories not shown in Fig. 19 tend to have characteristics intermediate to the 24-h bar charts. Cases belonging to the better separated groups (R-00h, R-24h, R-96h, and PRNR in Fig. 19) are more frequently correctly classified or classified within one to two 12-h groups of the correct classification group than those belonging to the less separated groups (R-48h and R-72h in Fig. 19). The R-36h (not shown), and to a lesser extent the R-48h through R-72h cases, are classified into all groups with nearly the same frequency, which reflects little ability to correctly distinguish the time to recurvature for these synoptic situations. Notice that only 28% (30%) of the R-48h (R-36h) cases were misclassified into straight-track groups.

2. Additional model output to assist the forecaster

The discriminant analysis model classifies an individual case into the group that has the highest classification function score (discussed in Section IV.A). Discriminant

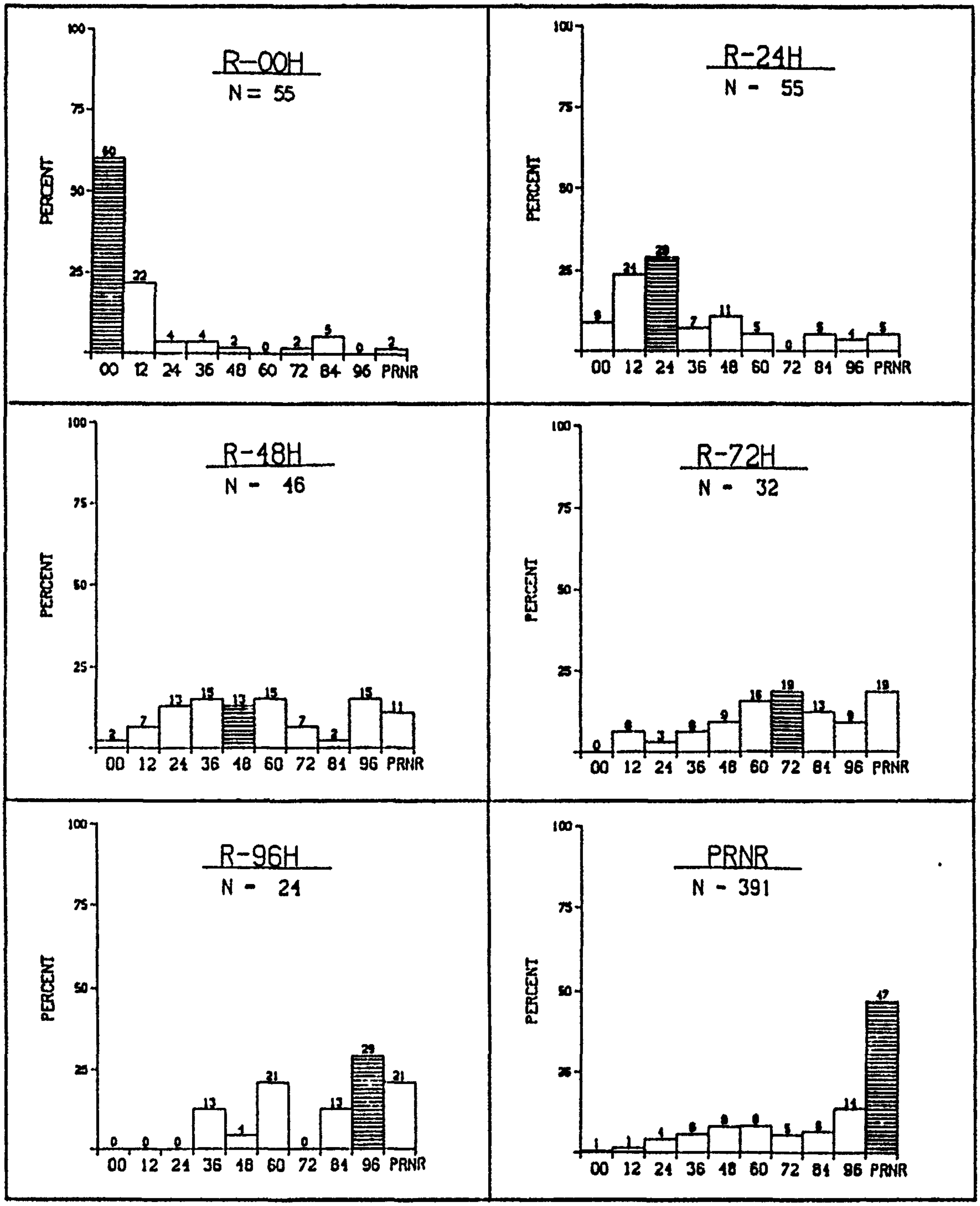

Fig. 19. Classification bar charts at 24-h intervals for the ten-group (seven-mode) model. Percent of N cases (ordinate) verifying as R-00h (top left), R-24h (top right), R-48h (middle left), R-72h (middle right), R-96h (bottom left), and PRNR (bottom right) that are classified into each group R-00h through PRNR (abscissa). Shaded bars indicate the percent in the correctly classified category.

analysis provides additional information that may assist the forecaster in subjectively assessing the validity of a model forecast. These outputs are the Mahalanobis distances and the posterior probabilities.

The Mahalanobis distance (D^2) is the squared distance of an individual case to each group centroid. Since D^2 has the same properties as the chi-squared (χ^2) statistic with degrees of freedom (*df*) equal to the number of predictors, Mahalanobis distances are measured in chi-square units.

The posterior probability is the probability that an individual case belongs to a group, which is calculated from D^2 by assuming the cases in each group are clustered around the centroid in a multivariate normal distribution and that every case belongs to one of the groups. Posterior probabilities are more useful to the forecaster because a set of nearly equal (small) percentage values for the time categories indicates the likely uncertainty in time to recurvature. Because posterior probabilites are used subjectively in this study, their contribution to forecast skill will not be evaluated in this section.

3. Comparison of discriminant analysis and Euclidean distance models

Classification skill for the final ten-group discriminant analysis model and the Euclidean distance model is compared in Table 18. Although the learning data sets differ, the skill scores reflect the ability of each model to forecast all 782 cases. As expected, the discriminant analysis model (line 1) outperforms the Euclidean distance model in all areas except %S (discriminant analysis = 66, Euclidean distance = 68). Because the learning set for the Euclidean distance model is comprised of only 161 cases, the results for this model (line 3) are predominantly independent test results. Thus, a more equitable comparison is between the jackknifed results for the discriminant analysis (line 2) and the Euclidean distance model. In this comparison, the discriminant analysis model still outperforms the Euclidean distance model in all areas except %S (jackknifed discriminant analysis = 65). The conclusion that discriminant analysis is a better method for exploiting the predictive capability of the EOF coefficients is based on relative performance of the two forecast models. The Euclidean distance method is an intuitive, and thus more subjective, method of forecasting tropical cyclone recurvature using EOF predictors of synoptic vorticity. Discriminant analysis is a statistically-based, and thus more objective, method for classifying the time to recurvature. Both analyses demonstrate skill compared to the climatological model (line 4) in which predictions are based on the relative historical frequency of occurrence of each 12-h classification group in straight and recurving best track data.

Table 18. **COMPARISON OF MODEL FORECAST SKILL:** Classification skill in terms of I-score, R-score and percent correctly classified recurver (R-00h to R-72h) or straight (R-84h to PRNR) for the final ten-group discriminant analysis model and the Euclidean distance model based on 250 mb vorticity EOF coefficients and climatological forecasts based on 1979-1984 best track data. Discriminant analysis jackknifed results (discussed in Section IV.B.1) reflect the skill expected with independent testing.

ANALYSIS METHOD	LEARNING SET	I-SCORE	R-SCORE	%R %S %T	MODES:
DISCRIMINANT ANALYSIS	ENTIRE SAMPLE	2.18	1.88	80 66 72	1 2 3 5 4 6 24
JACKKNIFED RESULTS		2.26	2.00	79 65 71	
EUCLIDEAN DISTANCE	CLEAN SET	2.34	2.10	75 68 71	1 6 10 12 15
CLIMATOLOGY	1979-1984	3.93	5.30	37 63 54	

F. VIOLATION OF ASSUMPTIONS

Although discriminant analysis is a robust procedure, the analysis results may be adversely affected by the violation of the requirements or assumptions to apply the method:

1. two or more distinct groups must be specified;
2. at least two cases must be present in each group;
3. the number of discriminating variables must be less than the total number of cases minus two;
4. the discriminating variables must be measured such that the differences between successive values are always the same;
5. the discriminating variables must not be a linear combination of the other discriminating variables;
6. the variance-covariance matrices must be approximately equal for each group; and
7. the group distributions must be multivariate normal.

Effects of violating the seven discriminant analysis assumptions are explained in detail in Klecka (1980), who notes that the best guide for a prediction model is the percentage of correct classifications. If the percentage is high, any violations were not harmful. If the percentage is low, it could be due to the violation of the assumptions or weak discriminating variables.

The first four assumptions are met by the data in this study. The BMDP7M program incorporates tolerance criteria in the stepwise selection of discriminating variables that protect against violations of multicollinearity (fifth assumption). Homogeneity of the variance-covariance matrices (sixth assumption) is more important in classification than in statistical inference. Cases tend to be over-classified into more disperse groups. Homogeneity of the variance-covariance matrices is tested by examination of the group standard deviations for each predictor and by inspection of the scatter plots of the first two canonical function scores for the cases in each group. The ten group standard deviations have no gross discrepancies in predictor variance. The largest differences in the variances are observed in Mode 1, and range from 39.2 for the R-00h group to 15.4 for the R-96h group. The canonical discriminant function scatter plots for each group (not shown) have roughly equal dispersion, which indicates that the variance-covariance matrices are approximately homogeneous.

Testing the multivariate normality (seventh assumption) of all linear combinations of the sample predictors is not currently feasible (Tabachnick and Fidell 1989). However, discriminant analysis is robust to violations of normality if they are caused by skewness rather than outliers. To test for outliers, the Mahalanobis distance from each group centroid to its member cases is evaluated as χ^2 with degrees of freedom equal to the number of predictors. Only three of the 782 cases in the sample population (Table 19) exceed the critical $\chi^2 = 24.32$ at $\alpha = 0.001$ with seven *df*. These three outliers are from recurving storms at R-00h or R-12h and two of them are from the Euclidean distance clean set storms (TY Vernon and ST Forrest). Eliminating the three multivariate outliers from the discriminant analysis (not shown) does not appreciably change the classification accuracy for the ten 12-h groups, regardless of whether the same seven predictors are hierarchically entered into the analysis or seven new predictors are selected in a stepwise fashion. However, the exclusion of these three cases from the sample population causes subtle changes in the F-to-enter statistics for each predictor. For example, the first seven modes selected in the stepwise analysis are Modes 1, 2, 3, 5, 4, 45, and 6 instead of Modes 1, 2, 3, 5, 4, 6, and 24. Such multivariate outliers should be eliminated from the analysis to develop an operational forecast model. Since one goal of this study is to compare the classification skill for the discriminant analysis model with the Euclidean distance model that was derived using two of these cases, the multivariate outliers are not excluded from the final discriminant analysis model.

Kachigan (1982) questioned whether discriminant analysis is an appropriate analysis technique for the dichotomization of a continuous criterion variable, such as time to

Table 19. **MULTIVARIATE OUTLIERS:** Cases for which the Mahalanobis distance to the group centroid exceeds the critical χ^2 value of 24.3 for the final discriminant analysis model.

STORM NO/YR	STORM NAME	VERIFICATION CATEGORY	MODEL FORECAST	MAHALANOBIS DISTANCE TO VERIFICATION GROUP CENTROID
1679	TY LOLA	R-00H	R-12H	25.7
2280	TY VERNON	R-12H	R-00H	27.9
1183	ST FORREST	R-00H	R-00H	41.1

recurvature in this study. Although the recurvature and non-recurvature samples represent distinct sets, the synoptic situations that lead to recurvature do evolve continuously in time and thus may not be easily distinguished. Regression analysis may be a more powerful and efficient analysis procedure since the regression method would fully utilize the time resolution of the observed data and the time trends in the EOF predictors to model the time to recurvature as a continuous variable. The ten-group discriminant analysis model also makes full use of the time resolution of the data, but without any data transformations that might be required to meet the linearity assumptions of the regression model. Thus, discriminant analysis provides an efficient first look at the ability of EOF coefficients of synoptic vorticity to predict time to recurvature.

V. FORECAST EXAMPLES

Operational application of a discriminant analysis model using EOF predictors to forecast tropical cyclone recurvature is relatively simple. Only a personal computer or programmable calculator would be required to interpolate the analyzed wind fields onto the storm-centered grid, compute the vorticity at the gridpoints, calculate the EOF eigenvalues corresponding to the vorticity field and to solve for the classification function scores and posterior probabilities for each of the model's classification groups. In this section, forecast examples from the learning set are presented. The use of posterior probabilities to assess the validity of a forecast is also discussed.

A. TEST CASES

The final discriminant analysis model forecasts are presented for the 1984 examples (Fig. 1) of a recurver (ST Vanessa), a straight-mover (TY Agnes) and an odd-mover (ST Bill). The forecast skill for these three storms is typical of other storms in the data set.

1. Recurver

The final discriminant analysis model forecasts of the time to recurvature for ST Vanessa are shown in Fig. 20. ST Vanessa tracked along the southern side of the subtropical ridge, which had redeveloped in the wake of TY Tad, for nearly five days before recurving (ATCR 1984). Only the two discriminant analysis model forecasts of R-72h for times greater than 96 h before recurvature are clearly erroneous. All forecasts within 72 h of recurvature are correct predictions of a recurver-track type. Although only three of the seven recurver-track type forecasts are correct (R-00h, R-48h and R-72h), the forecasts all progress in a sequential manner toward recurvature (R-72h, R-72h, R-48h, R-48h, R-36h, R-36h, R-00h). The 12-h forecast sequences for most of the recurving storms in the sample have a similar progression. Although a prediction may be repeated at successive 12-h forecasts and one or more sequential classification groups may be skipped between successive forecasts, the predictions tend correctly toward recurvature. Such a consistent trend toward recurvature in successive operational forecasts would add confidence to the individual 12-h recurving-track forecasts.

2. Straight-mover

The final discriminant analysis model forecasts for TY Agnes are presented in Fig. 21. TY Agnes tracked west-northwest under the influence of an easterly steering flow along the south side of a broad mid- to low-level subtropical ridge that extended

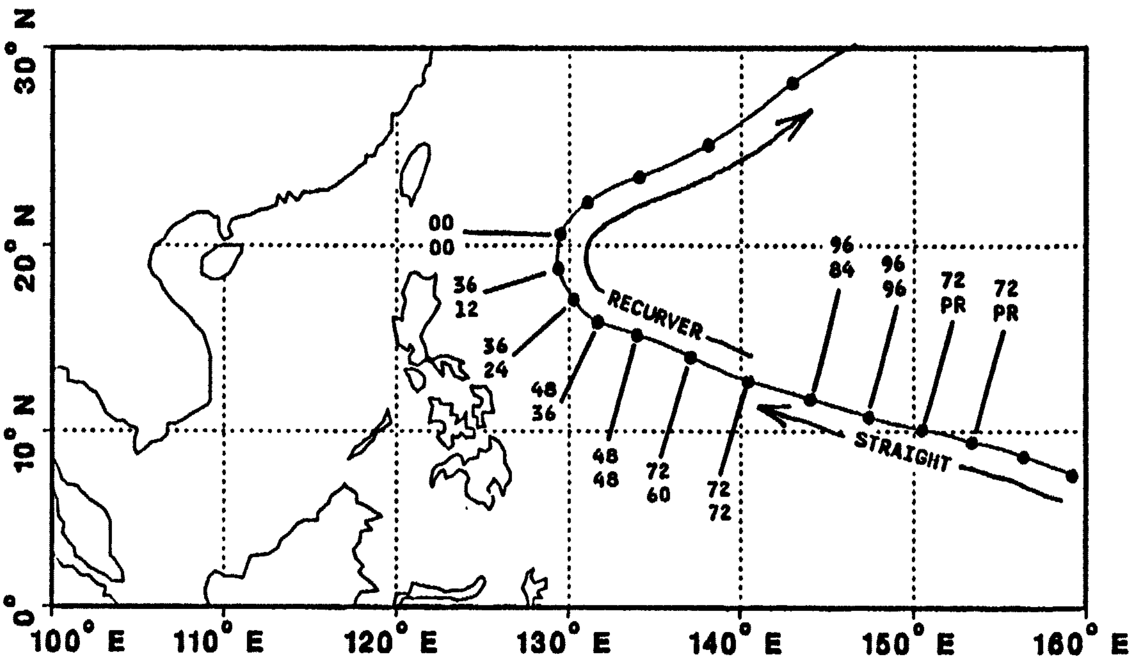

Fig. 20. Time-to-recurvature forecasts for recurving ST Vanessa. Discriminant analysis model forecasts (top number) and verifying time (bottom) to recurvature (h) at the JTWC best track 00 and 12 UTC positions during 22-31 October 1984 (dots). The letters PR indicate a pre-recurvature situation of more than 96 h prior to recurvature.

from the dateline west to the coast of Vietnam (ATCR 1984). Seven of the nine forecasts correctly predicted straight-track motion during the 72-h forecast period. Two forecasts of recurvature in 60 h are mispredictions of the track type. These two R-60h forecasts are 48 and 60 h (72 and 84 h) before landfall in Vietnam and subsequent dissipation.

3. **Odd-mover**

The forecast model in this study was not designed to distinguish odd-mover behavior such as loops and stairstep tracks. Therefore, forecasts based on the vorticity fields preceding or during erratic motion cannot provide accurate information on the storm's track. However, classifications may indicate storm motion if the next segment of the track fits either of the model's straight or recurver track categories.

The time-to-recurvature forecasts for ST Bill are shown in Fig. 22. Although ST Bill was expected to recurve similar to ST Vanessa, the complex environmental steering associated with an interaction with TY Clara caused Bill to track southeastward before dissipating east of the Philippines (ATCR 1984). In the first 48 h after the

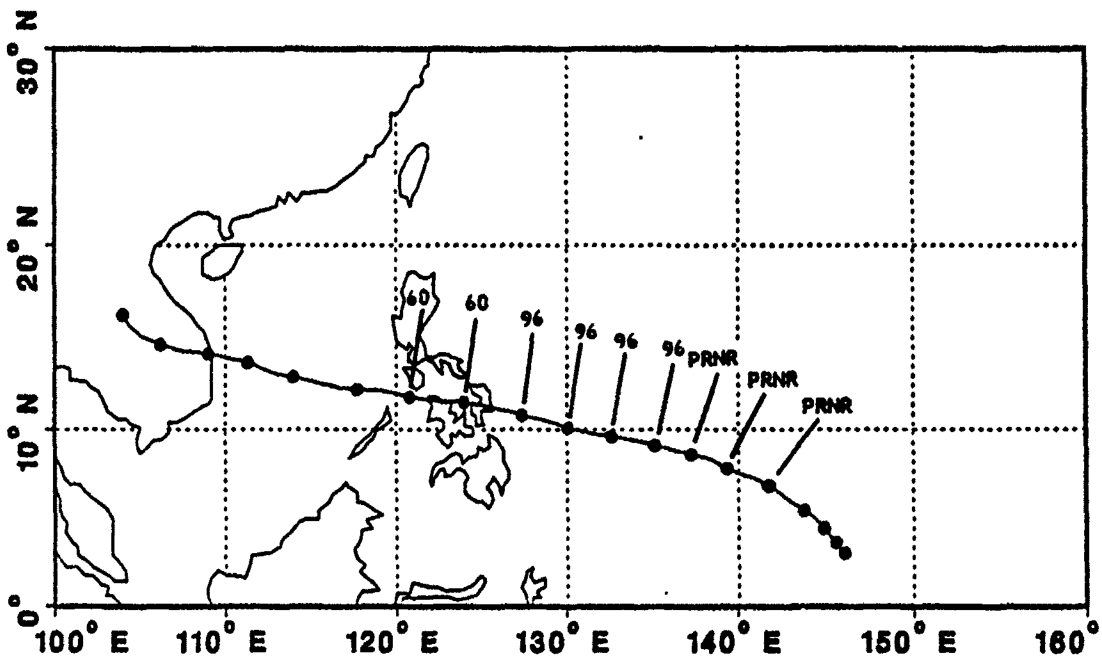

Fig. 21. Time-to-recurvature forecasts for TY Agnes. Discriminant analysis model forecasts of time to recurvature (h) at the JTWC best track 00 and 12 UTC positions (dots) during 1-8 November 1984. Definition as a straight-moving storm requires a minimum of 72 h after the forecast time to ensure verification as a straight-mover. PRNR refers to the forecast model classification group for recurving cases more than 96 h prior to recurvature time and straight-moving cases.

tropical cyclone formation alert (TCFA), Bill tracked slowly in a 25 n mi (46 km) diameter cyclonic loop. Although the next track segment is straight, forecasts during Bill's first loop predict recurvature in 60 h (first forecast) to 72 h (second through fourth forecasts). Once the erratic looping is completed, the model correctly identifies the straight-track segment in the next eight forecasts. As Bill began to recurve around the western end of the subtropical ridge, the midlatitude trough passed to the north and weakened the ridge, which slowed Bill's progress. The intense low-level circulation in the Philippine Sea associated with TY Clara, and the strengthening northeast monsoon flow, forced Bill to the southeast in an anticyclonic loop, and Bill rapidly weakened.

This set of model forecasts is unusual in that there is a sudden transition from straight-track predictions (R-96h) to the recurver predictions (R-24h, R-12h and R-00h). The model forecasts for recurving storms tend to transition more appropriately through successive recurvature classification categories. The model classifies the vorticity fields during the anticyclonic loop as recurvature (R-00h) situations. Since the forecast model

is unable to predict looping or southeast motion, synoptic situations after Bill's ..ould-be recurvature time (fifth R-00h forecast) are classified into the most similar of the ten straight plus recurver groups. While these recurvature forecasts correctly predict the recurvature-like motion as Bill moves northwest and then north and northeast, there is no indication in the model forecasts that the Bill will subsequently loop toward the southeast. The last two forecasts of R-12h and R-36h are based on the synoptic situation associated with Bill's southeast motion and precede a small cyclonic loop. These last forecasts indicate that the situation has changed, but continue erroneously to predict recurvature.

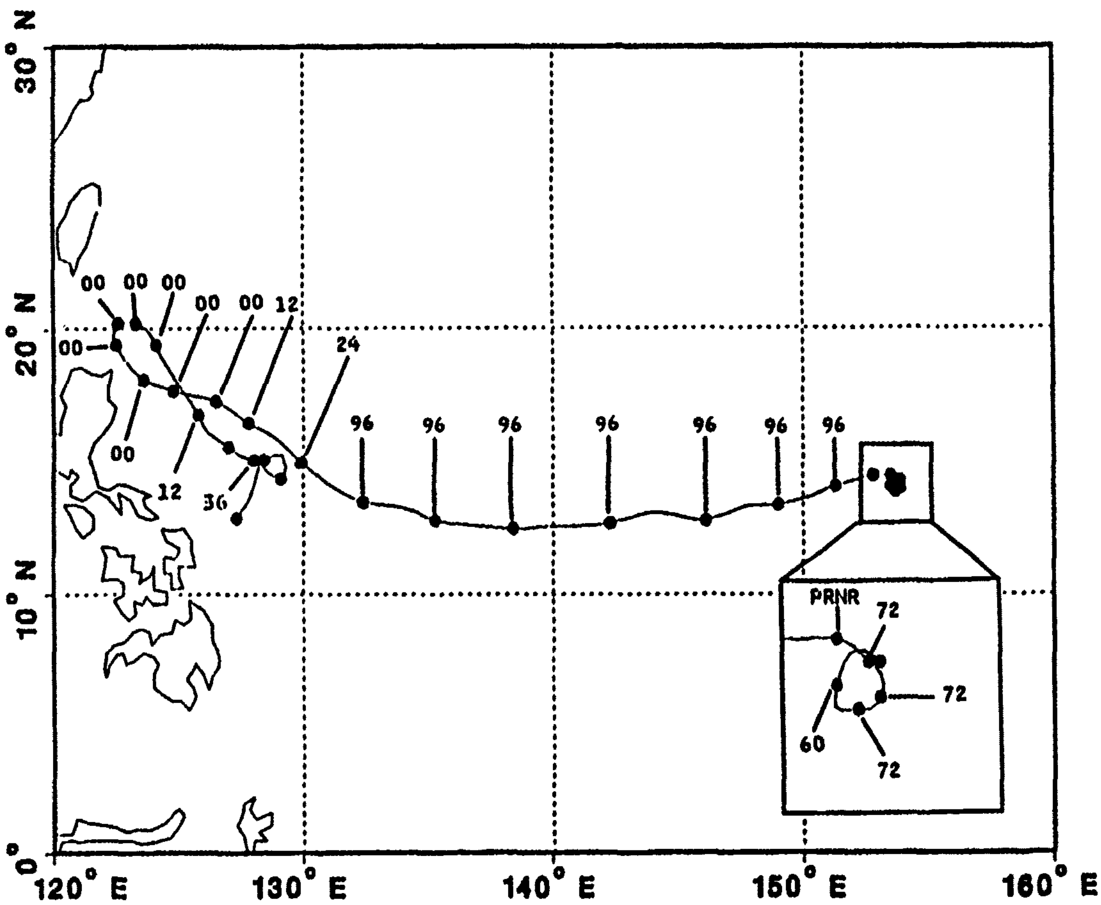

Fig. 22. Time-to-recurvature forecasts for ST Bill. Discriminant analysis model forecasts of time to recurvature (h) are indicated at the JTWC best track 00 and 12 UTC positions (dots) during 8-21 November 1984.

B. POSTERIOR PROBABILITIES AS AN AID IN THE FORECAST DECISION

The posterior probability is the probability that an individual case belongs to a group. The probabilities for all groups sum to one. The posterior probability (P) that

case i belongs to group j is computed from the Mahalanobis Distance (D^2) or directly from the classification function score (S) for the *ith* case for the *jth* group:

$$P_{ij} = \frac{\exp(S_{ij})}{\sum_{k=1}^{g} \exp(S_{ik})}. \tag{5.1}$$

Posterior probabilities can be used subjectively by the forecaster to assess the likelihood that a classification is correct. If the posterior probability for one classification group is high relative to the probabilities for the remaining groups, the forecaster can have more confidence that the model forecast is correct. If the posterior probability for the classification group is low and nearly equal to the probabilities for one or more of the other groups, then the forecaster should have less confidence in the prediction. Posterior probabilities can also be useful when a classification is repeated at successive 12-h forecasts to indicate whether the forecast is more or less likely to be correct.

The posterior probability would be more useful if some cutoff value existed that would indicate the forecast was likely to be correct. To examine whether this is the case for the discriminant analysis model, posterior probabilities for all cases classified into each 12-h forecast category are plotted as a function of the actual verification categories (Fig. 23). The ranges of the posterior probabilities vary with the forecast classification group. Probabilities are highest for the R-00h and PRNR forecasts and are lowest for the R-36h through R-84h forecasts. Unfortunately, the posterior probabilities are not distinctly higher for the correct predictions than for the incorrect predictions. Posterior probabilities for correct classifications are most distinct from incorrect classifications when PRNR is forecast. Therefore, posterior probabilities are most useful in evaluating PRNR forecasts.

Posterior probabilities for recurving storm ST Abby are presented in Table 20. ST Abby continually tracked to the right of the 1983 JTWC official forecasts (ATCR 1983). Although the JTWC forecast aids and numerical progs had consistently indicated a west-northwest track for Abby, the subtropical ridge over Japan never intensified as anticipated and Abby recurved to the northeast. Sandgathe (1987) cites ST Abby as an unusual example of a cyclone-subtropical ridge interaction, defined as a "through-the-ridge" case, in which the cyclone unexpectedly moves through an apparently well-established subtropical ridge.

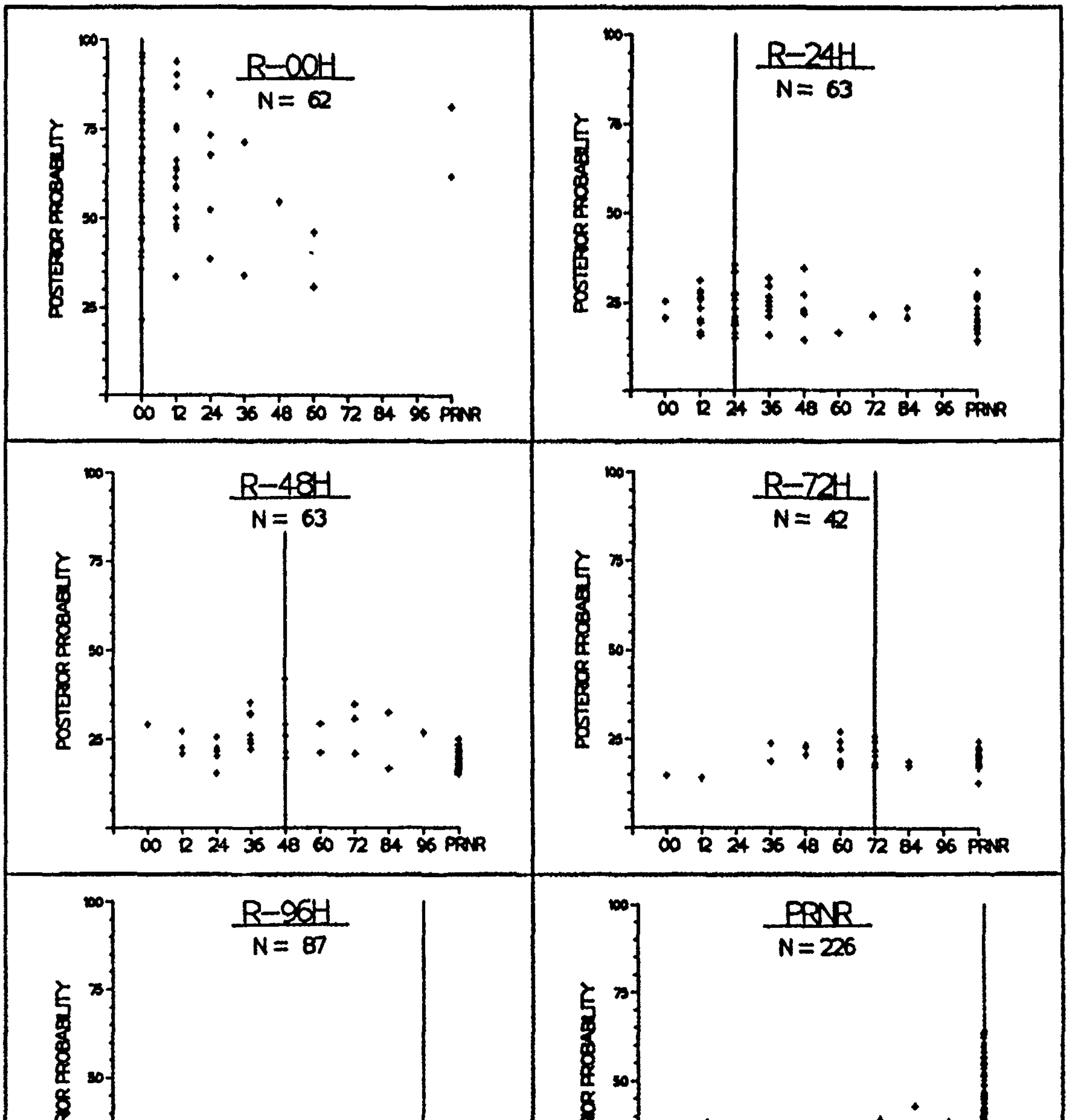

Fig. 23. Posterior probabilities of classifications into time-to-recurvature groups. Posterior probabilities (ordinate) for the N cases forecast as R-00h (top left), R-24h (top right), R-48h (middle left), R-72h (middle right), R-96h (bottom left), and PRNR (bottom right) plotted in the verifying groups R-00h through PRNR (abscissa). Vertical lines indicate the correct classifications.

On 5 August 1983, the discriminant analysis model correctly forecasts Abby's straight-track motion during the next 72 h and the posterior probability (35%) is relatively high. Referring to Fig. 23, only one case in the learning set had a PRNR forecast with a posterior probability greater than 35% and then recurved within the 72-h forecast period (R-12h). Therefore, the 35% posterior probability indicates that it is highly likely that the PRNR forecast is correct. Similarly, the second PRNR forecast has a relatively high posterior probability for the PRNR classification, which indicates the reliability of the PRNR forecast. Although the third PRNR forecast correctly predicts straight-track motion during the next 72-h period, the posterior probability that it belongs to that group is only 21%. Based on the learning set results in Fig. 23, a forecaster would have relatively less confidence in this PRNR forecast (line 3) than in the previous two PRNR forecasts (lines 1 and 2). However, the model continues to predict Abby as a straight-mover (or at least 84 h to recurvature) throughout the remainder of the recurvature period. The small posterior probability values indicate that the erroneous straight-track PRNR predictions are not likely to be correct.

Table 20. DISCRIMINANT ANALYSIS MODEL FORECASTS FOR ST ABBY: Month-day-times from 5-9 August 1983 are indicated in the DTG column. Verification times to recurvature are given in the VERF column. The prediction of the most likely classification group (time to recurvature in hours or PRNR) is based on the highest classification function score and corresponds to the highest posterior probabilities given in the columns labeled 00 through PRNR.

			MODEL CLASSIFICATION GROUP									
DTG	VERF	PRED	00	12	24	36	48	60	72	84	96	PRNR
080500	PRNR	PRNR	2	4	4	5	6	8	10	13	13	35
080512	96	PRNR	5	6	5	6	6	8	9	11	11	34
080600	84	PRNR	14	10	6	5	6	9	10	10	9	21
080612	72	PRNR	10	6	5	4	6	12	14	13	13	16
080700	60	PRNR	7	6	5	5	7	14	15	14	12	15
080712	48	PRNR	2	2	4	5	9	14	14	17	14	19
080800	36	84	1	2	5	8	9	10	12	19	15	18
080812	24	PRNR	0	2	5	7	7	8	11	21	17	21
080900	12	84	0	1	4	6	6	8	11	24	18	23
080912	00	84	1	2	4	6	7	12	16	22	15	15

VI. SUMMARY AND CONCLUSIONS

The feasibility of using an empirical orthogonal function (EOF) representation to identify the synoptic vorticity associated with tropical cyclone recurvature is examined. Recurvature, which is defined as a change in storm heading from west to east of 000 ° N, is evaluated from the Joint Typhoon Warning Center best track positions. In this EOF approach, the vorticity field is represented by the sum of 45 orthogonal eigenvectors that represent spatial patterns. Time-dependent coefficients are derived that indicate the importance of each pattern in the map series. The EOF coefficients are derived by Gunzelman (1990) from the 12-hourly U.S. Navy Global Band Analyses at 700, 400 and 250 mb for 1979-1984 western North Pacific tropical cyclones. The first 45 modes account for 73-78% of the variance in the relative vorticity fields.

The classification goals are two-fold: first, to identify tropical cyclone motion during the 72-h forecast period as either straight or recurving; and second, to forecast the time to recurvature with 12-h accuracy. The time series of the first and second EOF coefficients for recurving storms vary in a systematic manner as the tropical cyclone moves around the subtropical ridge. In contrast, the coefficients for straight-moving storms tend to cluster about different mean EOF 1-2 values. Taking this Euclidean distance approach, additional EOF predictors are identified that best separate recurvers and straight-movers in multidimensional EOF space. Classification of an individual case is then into the closest 12-h time-to-recurvature group or straight-mover category as measured in multidimensional EOF space. The Euclidean approach provides physical insight into the classification problem and demonstrates skill relative to climatological forecasts. However, there is no objective method of determining the optimum set of predictors or weighting the individual predictors in the model according to their significance is separating among the classification groups.

A more objective discriminant analysis technique is employed to more fully exploit the predictive capabilities of these EOF coefficients. In this approach, the entire set of 782 cases from 97 recurving and straight-moving tropical cyclones is used to both derive and test the recurvature model classifications. A final 250 mb discriminant analysis model is useful (72% correct) in identifying recurving (80%) and straight (66%) motion during the 72-h forecast period. Skill in distinguishing among the 12-h time to recurvature groups (R-00h through R-96h) plus the combined straight-mover and recurving

storm cases more than 96 h prior to recurvature (PRNR) is only 60, 29, 29, 12, 13, 22, 19, 7, 29, and 47%, respectively. While these results represent improvement over the Euclidean model forecasts, the skill in identifying the time to recurvature is less than desired for operational use. The relatively poor skill in classifying cases in the intermediate time to recurvature categories is attributed to the high variability among the synoptic fields that precede recurvature. Better skill (79% correct) in identifying storm motion during the 72-h forecast period can be achieved if classifications are only into two groups (recurver versus straight), rather than into the nine 12-h time-to-recurvature groups plus PRNR. Thus, the number and composition of the classification groups must be a trade-off between the forecaster's need to specify a precise time of recurvature versus the diminishing skill as more time precision is attempted in the forecast model.

The EOF coefficients for 250 mb vorticity provide the best time-to-recurvature forecast skill. The coefficients for this pressure level are statistically the most distinct among the time-to-recurvature groups and the 250 mb eigenvectors represent more variance in the vorticity fields than those for the other two pressure levels. In addition, the magnitude of the vorticity of the subtropical ridge increases with height and is greatest at 250 mb. The 700 mb coefficients provide the next best model skill. Although the eigenvectors for this pressure level account for less variance than those for 400 mb, the relative vorticity gradients between the cyclone and the subtropical ridge are greatest at 700 mb. Since more reliable data are available over open ocean areas at the upper levels from pilot reports and satellite-derived winds, the individual 12-hourly cases should be better defined and better forecast at 250 mb.

Since no classification groups are included for odd-mover motion, such as loops and stairsteps, these types of tracks are forecast into the most similar time-to-recurvature group. For example, an anticyclonic loop might be classified as recurvature. Perhaps the EOF representation of synoptic vorticity will not be able to identify the precise type of odd-mover motion resulting from the smaller and faster time scale forcing mechanisms such as multiple storm interactions. Thus, distinction between a storm that will merely step or loop to the northeast and one that will continue recurvature motion to the northeast is needed.

The results from these feasibility tests indicate the usefulness of an EOF representation of synoptic vorticity at one pressure level. Better skill may be achieved if the EOF coefficients for more than one pressure level are used, or if this EOF representation of the synoptic fields is combined with other factors such as persistence and climatology. Other analysis methods, such as multiple linear regression, that better exploit the time

trends in continuous data of this type should also be tested. As more data become available, independent testing and stratification of the sample will be possible. One problem with this initial investigation is that it is assumed that only one set of vorticity patterns leads to recurvature. In fr , several distinct paths may be defined by the time-dependent coefficients in multidimensional space. Such differences could be due to different forcing mechanisms associated with recurvature, or more simply due to the differences in the large-scale vorticity patterns with latitude. While these preliminary results in pinpointing the precise (12-h) time to recurvature are somewhat discouraging, other statistical techniques may prove more successful.

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